CALCULUS, Fourth Edition, Hughes-Hallett, Gleason, McCallum, et al.

Chapter 7:

Section	Suggested Problems
7.1	1, 3, 6, 9, 11, 14, 18, 19, 23, 27, 33, 34, 36, 37, 55, 59, 62, 65, 67, 73, 74, 76, 78-80, 85
7.2	1-27 (odd), 30, 31, 33, 34, 42, 43, 44, 52
7.3	1-37 (odd), 45
7.4	1, 3, 4, 5, 16, 17, 20, 23, 35, 45, 49, 57, 59, 62, 63
7.5	11, 13, 21, 22
7.6	1, 2, 8
7.7	1, 2, 5, 7, 13, 17, 19, 23, 29, 31, 33
7.8	1, 3, 12, 14, 20, 21, 24, 25, 35
Review	60, 85, 113, 135, 150, 151, 155

Problem. Find the indefinite integral $\int \sqrt{1-x^2}\,dx$.

Solution. First notice that

$$\frac{d}{dx}\left(\sqrt{1-x^2}\right) = -\frac{x}{\sqrt{1-x^2}}$$

Next we calculate,

$$\begin{aligned}
\int \sqrt{1-x^2}\,dx &= \int \frac{1-x^2}{\sqrt{1-x^2}}\,dx \\
&= \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{-x^2}{\sqrt{1-x^2}}\,dx \\
&= \arcsin x + \int x\,\frac{-x}{\sqrt{1-x^2}}\,dx \\
&= \arcsin x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx
\end{aligned}$$

1. Set $u = x,\ v' = \dfrac{-x}{\sqrt{1-x^2}}$
2. Use integration by parts
3. By the first line $v = \sqrt{1-x^2}$

We end with the same integral that we started with.

The last equality is in fact an equation which can be solved for $\int \sqrt{1-x^2}\,dx$. The solution is,

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\left(\arcsin x + x\sqrt{1-x^2}\right) + C.$$

Now we can calculate the definite integral

$$\int_0^x \sqrt{1-t^2}\,dt = \frac{1}{2}\left(\arcsin x + x\sqrt{1-x^2}\right).$$

It is remarkable that the last definite integral has a simple geometric interpretation. What is it?

The textbook suggests a different way to calculate this integral. Understand this alternative way as well and decide which one suits you better.