

Two problems related to Section 5.4

Problem 1. Consider the function

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0. \end{cases}$$

As it was discussed in Math 124, this function is continuous. Let n be a positive integer and consider the definite integrals

$$I_n = \int_0^{n\pi} f(x) dx, \quad n = 1, 2, 3, \dots$$

- (a) Use properties of the definite integral discussed in Section 5.4 to arrange the numbers $I_1, I_2, I_3, I_4, I_5, \dots$ in increasing order.
- (b) Explain your reasoning by stating explicitly which properties you use and how they apply to the definite integrals $I_1, I_2, I_3, I_4, I_5, \dots$
- (c) Do you recognize a pattern in the ordering of the numbers

$$I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, \dots ?$$

State this pattern clearly.

Problem 2. Consider the function $g(x) = (\sin x)^2$.

- (a) The function g has symmetries which can help you calculate the definite integrals below. Discover these symmetries and explain them.
- (b) Calculate the exact value of $\int_0^\pi g(x) dx$.
- (c) Calculate the exact value of $\int_0^{\pi/2} g(x) dx$.

Give detailed explanations of your reasoning.