I was asked to comment on the following integral. There probably are many ways to solve it this one seems efficient. Although, one might consider the first substitution somewhat obscure.

Integral 1. Find the integral below.

$$\int \frac{1}{x^2\sqrt{1+x^2}}\,dx =$$

$$w = \frac{1}{x} \quad \longleftarrow \quad \boxed{\begin{array}{l}\text{You might ask: why this substitution?}\\ \text{The answer is in the next calculation.}\\ \text{Assume that } x>0.\text{ For } x<0 \text{ calcualtions are similar.}\end{array}}$$

$$\frac{dw}{dx} = -\frac{1}{x^2}$$

$$\frac{1}{x}\,dx = -dw \quad \longleftarrow \quad \boxed{\text{Hence } dw \text{ is already in the integrand.}}$$

$$x = \frac{1}{w} \quad \longleftarrow \quad \boxed{\text{Always useful, } x \text{ in terms of } w.}$$

$$\begin{aligned}\sqrt{1+x^2} &= \sqrt{1+\frac{1}{w^2}}\\ &= \frac{w}{\sqrt{1+w^2}} \quad \longleftarrow \quad \boxed{\text{We need this to complete the substitution.}}\end{aligned}$$

$$= -\int \frac{w}{\sqrt{1+w^2}}\,dw \quad \longleftarrow \quad \boxed{\begin{array}{l}\text{This integral is also solved by substitution.}\\ \text{This time the substitution is more “natural”.}\end{array}}$$

$$=$$

$$z = 1 + w^2 \quad \longleftarrow \quad \boxed{\text{This is a “natural” substitution.}}$$

$$\frac{dz}{dw} = 2w$$

$$w\,dw = \frac{1}{2}\,dz \quad \longleftarrow \quad \boxed{\text{Hence } dz \text{ is already in the integrand.}}$$

$$\begin{aligned}&= -\frac{1}{2}\int \frac{1}{\sqrt{z}}\,dz \quad \longleftarrow \quad \boxed{\text{An easy integral.}}\\ &= -\sqrt{z} + C\\ &= -\sqrt{1+w^2} + C \quad \longleftarrow \quad \boxed{\text{Go back to } w.}\\ &= -\sqrt{1+\left(\frac{1}{x}\right)^2} + C \quad \longleftarrow \quad \boxed{\text{Go back to } x.}\\ &= -\sqrt{1+\frac{1}{x^2}} + C\\ &= -\frac{\sqrt{x^2+1}}{x} + C \quad \longleftarrow \quad \boxed{\text{Done! Celebrate!}}\end{aligned}$$