

Fall 2017 Math 138 Topics for the final exam

Preliminaries about functions. Know:

- Formal definition of a function.
- Formal definitions of injection, surjection and bijection; important examples of bijections and definitions of the important inverse functions (relevant material posted on September 28)
- The definition of the composition of functions and have understanding of the website post on October 4 related to this definition

Limits, continuity Know:

- The formal definition of limit at $+\infty$ posted on October 6 and how to apply this definition in simple examples (posts of October 6 and October 9)
- The ϵ - δ definition of limit at a point c and how to apply this definition in simple examples (post of October 9 and October 11)
- ϵ - δ definition of continuity of a function at a point and how to apply it in simple examples (post of October 16)
- The Intermediate Value Theorem, the Extreme Values Theorem and how to apply them in simple situations (post of October 17)
- Understanding of properties of continuous functions posted on October 18 and how to apply them in simple situations

Derivatives. Know:

- The formal definition of differentiability and derivative of a function and how to apply these definitions in simple examples (post of October 19)
- the concept of the tangent line to a graph and its connection to the definition of derivative and how to calculate tangent lines to simple graphs and use them to find approximate values of functions ("Line is better than Sine." Why?)
- Using tangent lines and normal lines to graphs to find specially positioned circles (October 19 and 20)
- Understanding of Fermat's Interior Extremum Theorem, Mean Value Theorem, their geometric meaning and significance (October 23 and 27)
- The definition of Lambert's W functions and how to use them to solve simple problems, posts of October 27 and 30.
- How to find higher order approximations and how to use them to get approximations for values of a function; how to find the osculating circle for a function at a point, posts of October 31 and November 2.
- How to solve optimization problems using properties of differentiable functions, post of November 3
- The geometric relationship between the derivative of a bijection and its inverse and how to use it to calculate derivatives of the inverse.
- How to do implicit differentiation and how to use it to analyze simple implicit equations, post of November 7
- How to construct parametric equations of simple planar curves, their tangent and normal lines, posts of November 8 and 9.

- How to calculate the envelope of a simple family of lines, post of November 13. In particular, how to construct a family of normals to a graph and calculate their envelope and the relationship of this curve to the evolute, post of November 14.

Integration. Know:

- The definition of a Riemann sum of a function, definitions of special Riemann sums: Left, Right, Middle, Lower, Upper, post of November 14
- The formal definition of a Riemann integrable function and the definite integral of a function on an interval $[a, b]$, post of November 15
- How to use the formal definition to prove that $f(x) = x$ is Riemann integrable on $[a, b]$, post of November 15
- How to use known areas to find definite integrals
- The concept of the average value of a function
- The formal statements of the Fundamental Theorem of Calculus and how to use it solve related problems, post of November 28
- How to use definite integrals to calculate lengths of graphs and curves given by their parametric equations, post of November 29
- How to use definite integrals to calculate areas enclosed by graphs and curves given by their parametric equations, post of November 30
- How to use definite integral to calculate volumes with known cross-sections, post of December 1 and 5.
- How to use definite integrals to calculate the surface area of surfaces of revolution of graphs and curves given by their parametric equations, post of December 4
- How to use the method of cylindrical shells to calculate volumes of solids of revolution, post of December 8