Fall 2019 Math 204 Topics for the second exam

1.9 Matrix of a linear transformations. Know:

- ➢ Definitions of **injection** (**one-to-one**) and **surjection** (**onto**) for linear transformations and the characterizations in Theorems 11 and 12. Exercises 23-30.

2.1 Matrix operations. ➢ Know how to add matrices and multiply matrices with a scalar and properties of these operations.

- ➢ Know how to multiply two matrices (the definition and row-column rule for the computation).
- ➢ Know properties of matrix multiplication.
- ➢ Know the content of the post on October 22: For a given matrix A, find the Reduced Row Echelon Form (RREF) of A, then form the matrix which consists of the pivot columns of A and the matrix which consists of the nonzero rows of the RREF of A. What is the product of these two matrices?
- ➢ The transpose of a matrix and its properties.

2.2 The inverse of a matrix. ➢ Know the definition of an invertible matrix and the definition of an inverse of a matrix.

- ➢ Know the easy inverses: 2×2 matrices, elementary matrices, product of invertible matrices.
- ➢ Know the algorithm for finding A^{-1} and its connection with elementary matrices, see the post on October 25.
- ➢ Based on the post on October 25 be able to write an invertible matrix as a product of elementary matrices.
- ➢ How to use inverse to solve the equation $A\mathbf{x} = \mathbf{b}$.
- ➢ Theorem 7 and its proof.

2.3 Characterization of invertible matrices. Know:

- ➢ The statement and the proof of the invertible matrix theorem. (See examples of the proofs that we did in class.)

2.8 Subspaces of $\mathbb{R}^n$. ➢ Know the definition of a subspace of $\mathbb{R}^n$.

- ➢ Know that Example 3 gives the most important example of a subspace.
- ➢ Know the definition of a basis of a subspace of $\mathbb{R}^n$.
- ➢ Know the definition of $\operatorname{Col} A$ (for a given $n\times m$ matrix A) and how to find a basis for $\operatorname{Col} A$.
- ➢ Know the definition of $\operatorname{Nul} A$ (for a given $n\times m$ matrix A) and how to find a basis for $\operatorname{Nul} A$.
- ➢ Know the definition of $\operatorname{Row} A$ (for a given $n\times m$ matrix A) and how to find a basis for $\operatorname{Row} A$. (see the post of November 1)
- ➢ Know the post of October 31.

2.9 Dimension and rank. ➢ Know the definition of the **dimension** of a subspace of $\mathbb{R}^n$.

- ➢ Know the definition of the rank of an $m\times n$ matrix A.
- ➢ Know that for an $m\times n$ matrix A the dimension of the column space of A **equals** the dimension of the row space of A.
- ➢ The rank theorem for an $m\times n$ matrix A: The dimension of the column space plus the dimension of the null space of A equals the number of columns in A.