```
<<DiscreteMath`Combinatorica`
```

Ex 11

```
2^8
```

256

Ex 12

```
Sum[2^k, {k, 0, 6}]
```

127

Ex 15

```
26^4 + 26^3 + 26^2 + 26^1 + 1
```

475255

Ex 16

```
26^4 - 25^4
```

66351

Ex 17

Inclusion-exclusion

```
128^5 - 127^5
```

1321368961

Ex 18

(a)

$$\text{Floor}\left[\frac{1000}{7}\right]$$

142

- **(b)**

```
Floor[1000/11]
```

90

```
Floor[1000/(7 11)]
```

12

```
142 - 12
```

130

```
Length[Select[Range[999], And[IntegerQ[#/7], Not[IntegerQ[#/11]]] &]]
```

130

- **(c)**

```
Floor[1000/(11*7)]
```

12

- **(d)**

```
Floor[1000/11] + Floor[1000/7] - Floor[1000/(11*7)]
```

220

- **(e)**

```
Floor[1000/11] + Floor[1000/7] - 2 Floor[1000/(11*7)]
```

208

- **(f)**

```
999 - (Floor[1000/11] + Floor[1000/7] - Floor[1000/(11*7)])
```

779

(g)

```
90 - 9 (* two digit numbers *) + 9*9*8 (* three digit numbers *)
```

729

```
Length[Select[Table[k, {k, 100, 999}], And[Length[Union[IntegerDigits[#]]] > 2] &]] +
 Length[Select[Table[k, {k, 10, 99}], And[Length[Union[IntegerDigits[#]]] > 1] &]]
```

729

(h)

```
Length[Select[Table[k, {k, 100, 999}],
   And[Length[Union[IntegerDigits[#]]] > 2, EvenQ[#]] &]] + Length[
  Select[Table[k, {k, 10, 99}], And[Length[Union[IntegerDigits[#]]] > 1, EvenQ[#]] &]]
```

369

```
8*8*4 (* three digits ending with 2,4,6,8 *) +
 9*8 (* three digits ending with 0 *) + 8*4
 (* two digits ending with 2,4,6,8 *) + 9 (* two digits ending with 0 *)
```

369

```
Length[Select[Table[k, {k, 100, 999}],
   And[Length[Union[IntegerDigits[#]]] > 2, OddQ[#]] &]] + Length[
  Select[Table[k, {k, 10, 99}], And[Length[Union[IntegerDigits[#]]] > 1, OddQ[#]] &]]
```

360

```
8*8*5 (* three digits ending with 1,3,5,7,9 *) +
 8*5 (* two digits ending with 1,3,5,7,9 *)
```

360

Ex 19

(a)

```
105 / 7
```

15

```
994 / 7
```

142

```
142 - 15 + 1
```

128

- **(b)**

450

- **(c)**

9

- **(d)**

100 / 4

25

996 / 4

249

249 - 25 + 1

225

900 - 225

675

- **(e)**

(999 - 102) / 3 + 1

300

996 / 12 - 108 / 12 + 1

75

300 + 225 - 75

450

- **(f)**

450

- **(g)**

300 - 75

225

(h)

```
75
```

```
75
```

21

(a)

```
10^3 - 10
```

```
990
```

(b)

```
5 * 10 * 10
```

```
500
```

(c)

```
9 + 9 + 9
```

```
27
```

23

```
3^50
```

```
717897987691852588770249
```

25

```
26 * 26 * 10000 + 100 * 26^4
```

```
52457600
```

27

```
(26 26 + 26^3) (100 + 1000)
```

```
20077200
```

■ 29

■ (a)

```
Length[CharacterRange["a", "z"]]
```

26

```
Length[Complement[CharacterRange["a", "z"], {"a", "e", "i", "o", "u"}]]
```

21

```
21^8
```

37822859361

■ (b)

```
21 20 19 18 17 16 15 14
```

8204716800

■ (c)

```
5 26^7
```

40159050880

■ (d)

```
5 25 24 23 22 21 20 19
```

12113640000

■ (e)

```
26^8 - 21^8
```

171004205215

■ (f)

Where is vowel? 1, 2, 3, 4, 5, 6, 7, 8

```
8*5*21^7
```

72043541640

■ (g)

Start with x

```
26^7
```

8031810176

Start with x no vowels

```
21^7
```

1801088541

```
26^7 - 21^7
```

6230721635

■ (h)

```
26^6 - 21^6
```

223149655

■ 31

■ (a)

```
0
```

■ (b)

```
5!
```

120

■ (c)

```
6 5 4 3 2
```

720

■ (d)

```
7 6 5 4 3
```

2520

■ 33

■ (a)

If n = 1, then 2.

If n = 2, then 2.

If n > 2, none

■ (b)

If n = 1, then 1.

If n = 2, then 1.

If n > 2, 2^{n-2}

■ (c)

If n = 1, then 0.

If n = 2, then 2.

If n > 2, then 2(n-1).

■ 37

If n is even, then 2^(n/2). If n is odd, then 2^((n+1)/2). The common answer is 2^Ceiling[n/2].

■ 38

■ (a)

```
9*8*7*6*5*6 (* bride is in the picture *)
```

```
90720
```

```
9*8*7*6*5*6 (* groom is in the picture *)
```

```
90720
```

(b)

```
10 * 9 * 8 * 7 * 6 * 5 (* all possible *)
```

151200

```
8 * 7 * 6 * 5 * 4 * 3 (* no bride no groom *)
```

20160

```
10 * 9 * 8 * 7 * 6 * 5 - 8 * 7 * 6 * 5 * 4 * 3 (* at least one of b or g *)
```

131040

```
2 * 90720 - 131040 (* both b and g *)
```

50400

```
50400 (* both b and g *) + (90720 - 50400) (* only b *) +
 (90720 - 50400) (* only g *) + 20160 (* neither b or g*)
```

151200

(c)

```
(* exactly one of b or g *)
```

```
(90720 - 50400) (* only b *) + (90720 - 50400) (* only g *)
```

80640

39

(a)

```
2 * 5 !
```

240

(b)

```
6 ! - 2 * 5 !
```

480

■ (c)

$$\frac{6\,!}{2}$$

```
360
```

■ 41

```
2^7 + 2^8 - 2^5
```

```
352
```

■ 42

The number of strings with exactly 5 consecutive 0s

$$e5 = (2^4) + (2^3) + (2^3) + (2^3) + (2^3) + 2^4$$

```
64
```

The number of strings with exactly 6 consecutive 0s

$$e6 = (2^3) + (2^2) + (2^2) + (2^2) + 2^3$$

```
28
```

The number of strings with exactly 7 consecutive 0s

$$e7 = (2^2) + (2^1) + (2^1) + 2^2$$

```
12
```

The number of strings with exactly 8 consecutive 0s

$$e8 = (2^1) + (2^0) + (2^1)$$

```
5
```

The number of strings with exactly 9 consecutive 0s

$$e9 = (2^0) + (2^0)$$

```
2
```

The number of strings with exactly 10 consecutive 0s

$$e10 = 1$$

```
1
```

The number of strings with at least 5 consecutive 0s

```
e5 + e6 + e7 + e8 + e9 + e10
```

```
112
```

The number of strings with at least 5 consecutive 1s is the same

The number of strings with at least 5 consecutive 0s or at least 5 consecutive 1s

```
2 (e5 + e6 + e7 + e8 + e9 + e10) - 2
```

```
222
```

Total

2^{10}

```
1024
```

```
Length[Select[Strings[{0, 1}, 10], Max[Map[Length, Split[#]]] > 4 &]]
```

```
222
```

■ 43

Total

```
2^8
```

```
256
```

Exactly three consecutive 0s

```
(* start with 0001, exclude the string end with 0000 *)
 (* all with 10001 *)
 (* end with 1000, exclude the string start with 0000 *)
 (* one s in 1 st and 2 nd, one s in 2 nd and 3 rd, one s in 1 st and third *)
```

ne3 = ((2^4 - 1)) + ((2^3) + (2^3) + (2^3) + (2^3)) + ((2^4 - 1)) - 3

```
59
```

```
Length[Select[Strings[{0, 1}, 8],
  (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] == 3) &]]
```

```
59
```

Exactly four consecutive 0s

```
ne4 = 2^3 + 2^2 + 2^2 + 2^2 + 2^3
```

```
28
```

```
Length[Select[Strings[{0, 1}, 8],
  (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] == 4) &]]
```

```
28
```

Exactly five consecutive 0s

```
ne5 = 2^2 + 2^1 + 2^1 + 2^2
```

```
12
```

Exactly six consecutive 0s

```
ne6 = 2 + 1 + 2
```

```
5
```

Exactly seven consecutive 0s

```
ne7 = 1 + 1
```

```
2
```

Exactly eight consecutive 0s

```
ne8 = 1
```

```
1
```

```
ne3 + ne4 + ne5 + ne6 + ne7 + ne8
```

```
107
```

```
(ne4 + ne5 + ne6 + ne7 + ne8)
```

```
48
```

```
(ne3 + ne4 + ne5 + ne6 + ne7 + ne8) + (ne4 + ne5 + ne6 + ne7 + ne8)
```

```
155
```

Eight strings are in both:

```
{0, 0, 0, 1, 1, 1, 1, 0}

{0, 0, 0, 1, 1, 1, 1, 1}

{0, 0, 0, 0, 1, 1, 1, 1}

{1, 0, 0, 0, 1, 1, 1, 1}
```

```
{1, 1, 1, 1, 0, 0, 0, 0}

{1, 1, 1, 1, 0, 0, 0, 1}

{1, 1, 1, 1, 1, 0, 0, 0}

{0, 1, 1, 1, 1, 0, 0, 0}

(ne3 + ne4 + ne5 + ne6 + ne7 + ne8) + (ne4 + ne5 + ne6 + ne7 + ne8) - 8

147

Map[{Length[#], First[#]} &, Split[{0, 0, 0, 1, 0}]]

{{3, 0}, {1, 1}, {1, 0}}

(Max[Map[Length, Select[Split[#], First[#] == 0 &]]] > 2) &[{0, 0, 0, 1, 0}]

True

Length[Select[Strings[{0, 1}, 8],
  (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] > 2) &]]

107

Length[Select[Strings[{0, 1}, 8],
  (Max[Map[Length, Select[Split[#], First[#] == 1 &]]] > 3) &]]

48

Length[Select[Strings[{0, 1}, 8],
  Or[(Max[Map[Length, Select[Split[#], First[#] == 0 &]]] > 2),
    (Max[Map[Length, Select[Split[#], First[#] == 1 &]]] > 3)] &]]

147

Length[Select[Strings[{0, 1}, 8],
  And[(Max[Map[Length, Select[Split[#], First[#] == 0 &]]] > 2),
    (Max[Map[Length, Select[Split[#], First[#] == 1 &]]] > 3)] &]]

8

Select[Strings[{0, 1}, 8], And[(Max[Map[Length, Select[Split[#], First[#] == 0 &]]] > 2),
   (Max[Map[Length, Select[Split[#], First[#] == 1 &]]] > 3)] &]

{{0, 0, 0, 0, 1, 1, 1, 1}, {0, 0, 0, 1, 1, 1, 1, 0},
 {0, 0, 0, 1, 1, 1, 1, 1}, {0, 1, 1, 1, 1, 0, 0, 0}, {1, 0, 0, 0, 1, 1, 1, 1},
 {1, 1, 1, 1, 0, 0, 0, 0}, {1, 1, 1, 1, 0, 0, 0, 1}, {1, 1, 1, 1, 1, 0, 0, 0}}
```

■ No consecutive 0s

```
Length[Select[Strings[{0, 1}, 8],
  (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] < 2) &]]
```

55

```
Table[Length[Select[Strings[{0, 1}, k],
   (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] < 2) &]], {k, 1, 15}]
```

{2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597}

■ At most consecutive 00s

```
Length[Select[Strings[{0, 1}, 8],
  (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] < 3) &]]
```

149

```
Table[Length[Select[Strings[{0, 1}, k],
   (Max[Map[Length, Select[Split[#], First[#] == 0 &]]] < 3) &]], {k, 2, 15}]
```

{4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, 10609}

```
Clear[sol, n];
```

```
sol[2] = 4; sol[3] = 7; sol[4] = 13;
sol[n_] := sol[n] = sol[n - 1] + sol[n - 2] + sol[n - 3]
```

```
Table[sol[k], {k, 2, 15}]
```

{4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, 10609}

Solve[$x^2 - x - 1 == 0$, x]

$$\left\{\left\{x \to \frac{1}{2}\left(1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(1+\sqrt{5}\right)\right\}\right\}$$

{r1, r2, r3} = Simplify[x /. Solve[$x^3 - x^2 - x - 1 == 0$, x]]

$$\left\{\frac{1}{3}\left(1+\left(19-3\sqrt{33}\right)^{1/3}+\left(19+3\sqrt{33}\right)^{1/3}\right),\right.$$
$$\frac{1}{6}\left(2+\left(-1-i\sqrt{3}\right)\left(19-3\sqrt{33}\right)^{1/3}+i\left(i+\sqrt{3}\right)\left(19+3\sqrt{33}\right)^{1/3}\right),$$
$$\left.\frac{1}{6}\left(2+i\left(i+\sqrt{3}\right)\left(19-3\sqrt{33}\right)^{1/3}+\left(-1-i\sqrt{3}\right)\left(19+3\sqrt{33}\right)^{1/3}\right)\right\}$$

```
r1
```

$$\frac{1}{3}\left(1+\left(19-3\sqrt{33}\right)^{1/3}+\left(19+3\sqrt{33}\right)^{1/3}\right)$$

```
Simplify[Im[x /. Solve[x^3 - x^2 - x - 1 == 0, x]]]
```

$$\left\{0,\ \frac{-\left(19-3\sqrt{33}\right)^{1/3}+\left(19+3\sqrt{33}\right)^{1/3}}{2\sqrt{3}},\ \frac{\left(19-3\sqrt{33}\right)^{1/3}-\left(19+3\sqrt{33}\right)^{1/3}}{2\sqrt{3}}\right\}$$

```
nn = 2; Chop[NSolve[{x (r1) + y (r2) + z (r3) == 7,
   x (r1)^2 + y (r2)^2 + z (r3)^2 == 13, x (r1)^0 + y (r2)^0 + z (r3)^0 == 4}, {x, y, z}]]
```

```
{{x → 3.84797, y → 0.0760144 + 0.0113183 i, z → 0.0760144 - 0.0113183 i}}
```

```
Chop[Table[(x (r1)^k + y (r2)^k + z (r3)^k) /.
   {x → 3.8479711061989845`, y → 0.07601444690050753` + 0.011318308963751147` i,
    z → 0.07601444690050753` - 0.011318308963751146` i}, {k, 0, 12}]]
```

```
{4., 7., 13., 24., 44., 81., 149., 274., 504., 927., 1705., 3136., 5768.}
```