5

Job 1 goes to one of five, Job 2 goes to one of five, Job 3 goes to one of five. By product rule 5*5*5 = 125

Problem 20

```
Binomial[11 + 3, 3]
```

```
364
```

```
Length[Select[
  Flatten[Table[{x, y, z}, {x, 0, 11}, {y, 0, 11}, {z, 0, 11}], 2], (Apply[Plus, #] < 12) &]]
```

```
364
```

23

```
(* We are asked how many teams of two can be
  formed by 12 students. The teams have different colors. *)
```

```
(* There are two ways to think about this. First,
 team the students following the permutations. But,
 this would count each team twice. *)
```

$$\frac{12!}{(2!)^6}$$

```
7484400
```

```
(* The second way is by product
  rule: There are 12 choose 2 ways of selecting the red team,
 there are 10 choose 2 ways of selecting green team,
 there are 8 choose 2 ways of selecting blue team, ... *)
```

```
Product[Binomial[2 k, 2], {k, 6, 1, -1}]
```

```
7484400
```

Problem 25

```
Binomial[24, 5] - 6 Binomial[14, 5]
```

```
30492
```

```
Length[Select[IntegerDigits[#] & /@ Range[999999], (Apply[Plus, #] == 19) &]]
```

30492

How about the sum is 20

```
Length[Select[IntegerDigits[#] & /@ Range[999999], (Apply[Plus, #] == 20) &]]
```

35127

```
Binomial[25, 5] - (6 Binomial[15, 5] - Binomial[6, 2])
```

35127

How about the sum is 21

```
Length[Select[IntegerDigits[#] & /@ Range[999999], (Apply[Plus, #] == 21) &]]
```

39662

```
Binomial[26, 5] - (6 Binomial[16, 5] - 2 Binomial[6, 2])
```

39602

25

```
(* this is a wallet problem, but we are permited to put only digits 0,1,...,
 9 in six boxes. First we calculate all possibilities with nonnegative integers,
 then subtract those that contain 10 or larger integers. Fortunately
  10 or larger number can appear only at one spot *)
```

```
Binomial[24, 5] - 6 Binomial[14, 5]
```

30492

26

```
(* total sum to 13 *)
```

```
Binomial[13 + 5, 5] - 6 Binomial[3 + 5, 5]
```

8232

```
(* no 9 s *)
```

```
Binomial[13 + 5, 5] - 6 Binomial[4 + 5, 5]
```

7812

```
(* the difference is one nine *)

(Binomial[13 + 5, 5] - 6 Binomial[3 + 5, 5]) - (Binomial[13 + 5, 5] - 6 Binomial[4 + 5, 5])

420

(* different logic how meny with only 9 at first *)

6 Binomial[4 + 4, 4]

420

Length[
 Select[IntegerDigits[#] & /@ Range[999999], (And[Apply[Plus, #] == 13, Max[#] == 9]) &]]

420
```

27

```
Binomial[50 + 9, 9]

12565671261
```

33

```
{O, R, O, N, O}

Binomial[3, 1] (* one letter *) + (3 * 2 + 1) (* two letters *) +
 (3 * 2 * 1 + Binomial[3, 2] * 2 + 1) (* three letters *) +
 ( Binomial[4, 2] * 2 + Binomial[4, 3] * 2 ) (* four letters *) + 5!/3!

63
```

Problem 33

```
3 (* words with one letter *) +
 (* words with two letters *) +
  1 (* two Os *) +
 2 * 2 (* one O *) +
 2 (* no Os *) +
 (* words with three letters *) +
  1 (* three Os *) +
 Binomial[3, 2] * 2 (* two Os *) +
 Binomial[3, 1] * 2 (* one O *) +
 (* words with four letters *) +
   Binomial[4, 3] * 2 (* three Os *) +
 Binomial[4, 2] * 2 (* two Os *) +
 (* words with five letters *)
 Binomial[5, 3] 2
```

```
63
```

```
5! / 3!
```

```
20
```

```
Binomial[5, 3] Binomial[2, 1]
```

```
20
```

34

How many strings of five and more characters can be formed from the letters

```
{s, e, e, r, e, s, s}
```

using 7 characters

$$\frac{7!}{3!\,3!}$$

```
140
```

Using 6 characters we have the same number of strings since droping the last letter is a bijection between the set of all strings with 7 characters and the set of all strings with 6 characters

However, we can proceed and count them differently: (we use all but, s, e, r, respectively)

$$\frac{6!}{2!\,3!}+\frac{6!}{2!\,3!}+\frac{6!}{3!\,3!}$$

140

Using five characters we have to go through all the cases: drop {e,e}, drop {e,r}, drop {e,s}, drop {r,s}, drop {s,s}

$$\frac{5!}{1!\,3!}+\frac{5!}{2!\,3!}+\frac{5!}{2!\,2!}+\frac{5!}{3!\,2!}+\frac{5!}{3!\,1!}$$

90

From the strings with 5 characters we can calculate the number of strings with 7 characters counting the number of possible completions:

$$\frac{5!}{1!\,3!}*1+\frac{5!}{2!\,3!}*2+\frac{5!}{2!\,2!}*2+\frac{5!}{3!\,2!}*2+\frac{5!}{3!\,1!}*1$$

140

39

We have to make 12 steps: 4 in x drection, 3 in y direction and 5 in z direction. This is exactly the same as counting the number of strings of 12 characters {x, x, x, x, y, y, y, z, z, z, z, z}

$$\frac{(4+3+5)!}{3!\,4!\,5!}$$

27720

$$\frac{(3+3+3)!}{3!\,3!\,3!}$$

1680