MATH 209

Examination 1
April 19, 2012

Name ______________________

1. Let p and q be propositions.

   (a) Prove that the negation of $p \oplus q$ is $p \leftrightarrow q$.

   (b) Consider the proposition $(p \to q) \oplus (q \to p)$. Find an equivalent, but much simpler proposition. (You can use the truth table of the given proposition to discover an equivalent much simpler proposition.)

2. (a) State the definition of a rational number. State the definition of an irrational number.

   (b) Prove or disprove the following theorem: If a is rational and b is irrational, then ab is irrational.

3. The universe of discourse in this problem is the set of all real numbers.

   Consider the following proposition:

   "For every x there exists y such that for all z we have $z < y$ implies $z^2 > x^2$."

   (a) Write the given proposition using quantifiers.

   (b) State the negation of the proposition in (3a).

   (c) Decide which proposition is true: the proposition in (3a) or the proposition in (3b). Prove your claim.

4. Let A and B be sets. The set $A \oplus B$ is defined as $A \oplus B = \big\{x \mid (x \in A) \oplus (x \in B)\big\}$.

   (a) Let A, B, C be given sets. Use a Venn diagram to represent the set $\big(A \oplus B\big) \oplus C$.

   (b) Find a formula for the set represented by the Venn diagram in Figure 1. This formula should include the sets A, B and C. For the full credit you must use the set $\big(A \oplus B\big) \oplus C$ studied in (4a).

5. What is wrong with the proof given in the box below? Please be specific.

$$\frac{25}{36} = \frac{9+16}{36} \qquad (1)$$

$$\frac{25}{36} = \frac{1}{4} + \frac{4}{9} \qquad (2)$$

$$\frac{1}{36} = \frac{1}{4} - 2\frac{1}{2}\frac{2}{3} + \frac{4}{9} \qquad (3)$$

$$\left(\frac{1}{6}\right)^2 = \left(\frac{1}{2} - \frac{2}{3}\right)^2 \qquad (4)$$

$$\frac{1}{6} = \frac{1}{2} - \frac{2}{3} \qquad (5)$$

$$1 = 3 - 4 \qquad (6)$$

$$1 = -1 \qquad (7)$$

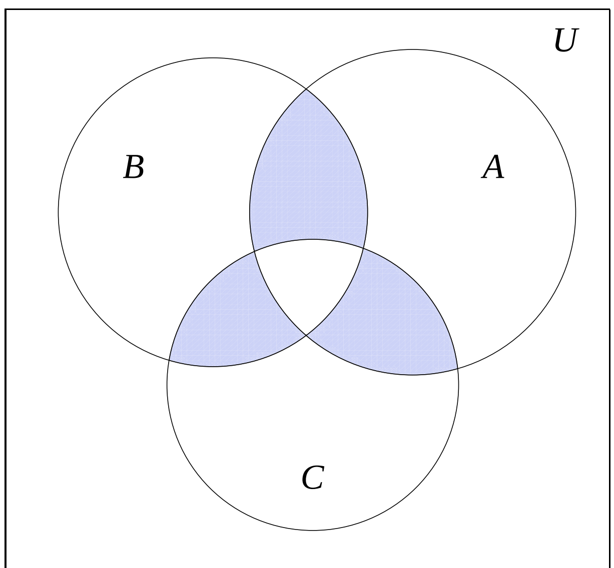

Figure 1: Venn diagram for (4b)