

This problem is inspired by Problem 37 in Section 13.1 and the Wikipedia page about hexagonal lattice.

There are two unit vectors in the first and second picture below. They are:

$$\mathbf{u} = -\mathbf{i} \quad \text{and} \quad \mathbf{v} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}.$$

The vector \mathbf{u} is the black vector and \mathbf{v} is the gray vector.

1. This item relates to the first picture. I claim that the position vector of each point in the first picture can be expressed as a sum of integer multiples of the vectors \mathbf{u} and \mathbf{v} . I know that this is true since that is how I created the picture. Select several specific points (the more points the better) in this picture and confirm that my claim is true for those points by giving the specific integers in question.
2. In the second picture I colored the points from the first picture red, green and blue according to a certain pattern. Discover the pattern that I used for coloring. Your explanation of the pattern must be clear and must involve the integers that were mentioned in the first item of this problem.
3. I created the third picture by using the coloring from the second picture and the vectors \mathbf{u} and \mathbf{v} and a third vector. Do you see which third vector I used and how? Please explain clearly.
4. Again, I created the fourth picture by using the coloring from the second picture and a certain pattern. Do you see the pattern that I used to color the hexagons. The pattern is similar to the pattern discovered in the second item. The explanation must involve the integers from the first item. Explain clearly.







