For most functions f a proof of $\lim_{x \to +\infty} f(x) = L$ based on the definition in the notes should consist from the following steps.

(1) Find X_0 such that $f(x)$ is defined for all $x \geq X_0$. Justify your choice.

(2) Use algebra to simplify the expression $|f(x) - L|$ with the assumption that $x \geq X_0$. Try to eliminate the absolute value.

(3) Use the simplification from (2) to discover the BIN:

$$|f(x) - L| \leq b(x) \quad \text{valid for} \quad x \geq X_0.$$

Here $b(x)$ should be a simple function with the following properties:

(a) $b(x) > 0$ for all $x \geq X_0$.

(b) $b(x)$ is tiny for huge x.

(c) $b(x) < \epsilon$ is easily solvable for x for each $\epsilon > 0$. The solution should be of the form

$$x > \quad \text{some expression involving } \epsilon.$$

(4) Use $\boxed{\text{the solution}}$ of $b(x) < \epsilon$ and X_0 to define $X(\epsilon) = \max\left\{X_0, \boxed{\text{the solution}}\right\}$.

(5) Use the BIN to prove the implication $\quad x > X(\epsilon) \Rightarrow |f(x) - L| < \epsilon$.