

Problem 1. Give symbolic (that is exact, not approximate) expression for the distance between the numbers

$$3e^{5\pi} + \pi^{-2e} \quad \text{and} \quad 19906872.$$

Do not use the absolute value in your answer. Explain your reasoning.

Problem 2. There is a simple formula expressing the sign function in terms of the unit step function. Can you discover this formula? Remember

$$\text{sign}(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases} \quad \text{and} \quad \text{us}(x) := \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$$

Hint:

Problem 3. (I) Consider the function

$$f(x) = |\text{sign}(x)|.$$

- (a) Determine the domain and the range of this function.
- (b) Sketch a detailed graph of this function.

(II) Consider the function

$$f(x) = \text{sign}(\sin(\pi x)).$$

- (a) Determine the domain and the range of this function.
- (b) Sketch a detailed graph of this function.
- (c) Give formulas for all important points on this graph.

Problem 4. Consider the function

$$f(x) = x \left\lfloor \frac{1}{x} \right\rfloor.$$

- (a) Determine the domain and the range of this function.
- (b) Sketch a detailed graph of this function (as detailed as possible by hand).
- (c) Give formulas for all important points on this graph.

Problem 5. Let $x, y \in \mathbb{R}$. Prove that $||x| - |y|| \leq |x - y|$.