MATH 226 Assignment 1b April 17, 2009 Hints

Hint for Problem 1: A detailed graph of the function $x\left\lfloor \frac{1}{x} \right\rfloor$ indicates the inequalities

$$\text{for } x>0 \text{ we have} \quad \text{something simple} < x\left\lfloor \frac{1}{x} \right\rfloor \leq \text{something simple},$$
$$\text{for } x<0 \text{ we have} \quad \text{something simple} \leq x\left\lfloor \frac{1}{x} \right\rfloor < \text{something simple}.$$

These inequalities can be proved using $u-1 < \lfloor u \rfloor \leq u$ which is proved in the notes. The above inequalities can be used to prove

$$\bigl| x\lfloor 1/x \rfloor - 1 \bigr| < \text{something simple}.$$

Hint for Problem 2: Playing "pizza-party" you can get an inequality

$$\frac{(\sin x)^2}{x\,(\sin x)^2+1} \leq \text{something simple}.$$

But, just to be on the safe side, this inequality. (Cross multiplying can give you an idea for a proof.)

Hint for Problem 3: Playing "pizza-party" will not work here. Here you need to use your calculator to guess

$$\frac{|\sin x|}{x\,(\sin x)^2+1} \leq \boxed{\begin{array}{l}\text{something simple, similar, but}\\ \text{not identical to Problem 2}\end{array}}.$$

With the good guess you should be able to prove this inequality.

Hint for Problem 4: Imitate the proofs from the notes on page 10. Use the points where $\sin(x)=0$, and the points where $(\sin x)^2=1$, similar to that proof. To prove that this function does not converge to any limit L consider two or three cases, as in the notes in Example 3.3.3. If you have problems proving that this function does not converge to any limit L, then prove that it does not converge to 1 and that it does not converge to 0.

Hint for Problem 5: The estimates from the original hint should look like: For $v>1$ we have

$$\text{something simple} \leq \ln v \leq \text{something simple}.$$

Set $f(x) = \ln\Bigl(\bigl(1+\frac{1}{x}\bigr)^x\Bigr)$. Simplify this expression using logarithm rules. Then substitute $v = 1+1/x$ in the above inequalities. Since $x>0$, you can now get estimates for $f(x)$:

$$\text{something simple} \leq f(x) \leq \text{something simple}.$$

These inequalities can be used to obtain

$$\bigl| f(x) - L \bigr| \leq \text{something simple}.$$