

Problem 1. Prove

$$\lim_{x \rightarrow 0} x \left\lfloor \frac{1}{x} \right\rfloor = 1.$$

Problem 2. Use the definition of limit to prove that

$$\lim_{x \rightarrow +\infty} \frac{(\sin x)^2}{x (\sin x)^2 + 1} = ?.$$

Problem 3. Use the definition of limit to prove that

$$\lim_{x \rightarrow +\infty} \frac{\sin x}{x (\sin x)^2 + 1} = ?.$$

Problem 4. Determine whether the function

$$x \mapsto \frac{1}{x (\sin x)^2 + 1}$$

has a limit when $x \rightarrow +\infty$. State your conclusion and justify it.

Problem 5. Guess the limit of the function $x \mapsto \ln\left(1 + \frac{1}{x}\right)^x$ as x approaches $+\infty$. Use the definition of limit to prove your guess.

Hint: You can get useful estimates for the logarithm function by approximating the gray area in the figure below by a smaller and a bigger rectangle.

