

Winter 2012 Math 226 Topics for the final

Limits. Know:

- definition and properties of absolute value function with rigorous proofs, avoiding cases whenever possible;
- fifteen different definitions of limits;
- how to prove limits based on definitions;
- how to prove that limit does not exist in simple cases;
- two versions of the squeeze theorem with proofs;
- how to use geometric arguments and squeeze theorems to formally prove three trigonometric limits and one logarithmic limit.

Continuity. Know

- ϵ - δ definition of continuity;
- how to use ϵ - δ definition of continuity to prove that some simple functions are continuous;
- how to use a geometric argument to prove that sine and cosine are continuous functions;
- A rigorous proof that a composition of continuous functions is continuous.

Sequences. Know

- definitions of convergence, boundedness, monotonicity;
- how to use the definition of convergence to prove that some simple sequences converge or diverge;
- relationships between convergence and boundedness with the corresponding proof;
- both versions of the Completeness axiom;
- monotone convergence theorem and its proof;
- proofs related to sequences defined by a simple formula (Theorems 7.2.4 and 7.2.5);
- proof that the sum of two convergent sequences is a convergent sequence.

Infinite series. Know

- definition of convergence for series;
- all about geometric series with proofs;
- application of geometric series to decimal expansions with the proof of convergence of decimal expansions;
- a rigorous proof that the harmonic series diverges;
- how to recognize telescopic series and prove its convergence;
- a very important sufficient test for convergence with the proof (Theorem 8.2.1);
- how to apply the algebra of convergent series to determine convergence or divergence of series;
- how to apply comparison test and integral test to determine convergence of various series with estimates of the sum (p -series is an important application);
- how to apply the alternating series test to determine convergence of alternating series;
- the concept of absolute and conditional convergence;
- how to prove that each absolutely convergent series converges;
- how to apply the ratio and the root test to determine absolute convergence of series;

Power series. Know

- how to determine the interval of convergence of a given power series, in particular how to decide whether the endpoints of the interval belong to the interval of convergence;
- how to apply Theorem 9.2.1 to find sums of some simple power series.