MATH 226 Assignment 1 June 26, 2014

Name ______________________

Problem 1. There is a simple formula expressing the sign function in terms of the unit step function. Can you discover this formula? Remember

$$\operatorname{sign}(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases} \qquad \text{and} \qquad \operatorname{us}(x) := \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$$

We are seeking an explicit, not a piecewise formula. Prove the formula that you discovered.

Problem 2. Prove that for all real numbers x and y we have

$$\max\{x, y\} = \frac{1}{2}\big(x + y + |x - y|\big).$$

Discover and prove the analogous formula for the minimum.

Problem 3. (I) Consider the function

$$f(x) = \big|\operatorname{sign}(x)\big|.$$

(a) Determine the domain and the range of this function.

(b) Sketch a detailed graph of this function. Follow the logic of the graphs of sign, us, floor and ceiling and mark some points by disks (•) and some by circles (∘).

(c) Give formulas for <u>all</u> points which you marked by disks and all points that you marked by circles.

(II) Consider the function

$$f(x) = \operatorname{sign}(\sin(\pi x)).$$

(a) Determine the domain and the range of this function.

(b) Sketch a detailed graph of this function. Follow the logic of the graphs of sign, us, floor and ceiling and mark some points by disks (•) and some by circles (∘).

(c) Give formulas for <u>all</u> points which you marked by disks and all points that you marked by circles.

Problem 4. Consider the function

$$f(x) = x \left\lfloor \frac{1}{x} \right\rfloor.$$

(a) Determine the domain and the range of this function.

(b) Sketch a detailed graph of this function (as detailed as possible by hand). Follow the logic of the graphs of sign, us, floor and ceiling and mark some points by disks (•) and some by circles (∘).

(c) Give formulas for <u>all</u> points which you marked by disks and all points that you marked by circles.

Problem 5. Let $x, y, z \in \mathbb{R}$. Prove the following inequalities

(a) $|x - y| \leq |x - z| + |z - y|$.

(b) $\big||x| - |y|\big| \leq |x - y|$.

Problem 6. Prove that for all $x \in \mathbb{R}$ we have

$$\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \tfrac{1}{2} \rfloor.$$

Discover and prove the analogous identity for the ceiling function.