MATH 226 Assignment 3 February 12, 2016 Name ______________________

Problem 1. (a) Prove the following theorem.

Theorem. *Let f be a function which is defined on the interval $[1, +\infty)$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{x\to+\infty} f(x) = L$, then $\lim_{n\to+\infty} a_n = L$.

(b) Is the converse of the above theorem true? That is, is the following theorem true:

Theorem. *Let f be a function which is defined on the interval $[1, +\infty)$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{n\to+\infty} a_n = L$, then $\lim_{x\to+\infty} f(x) = L$.

Justify your answer.

Problem 2. (a) Prove the following theorem.

Theorem. *Let f be a function which is defined on the interval $(0, 1]$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(1/n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{x\downarrow 0} f(x) = L$, then $\lim_{n\to+\infty} a_n = L$.

(b) Formulate the converse of the above theorem. Is the converse true? Justify your answer.

Problem 3. Let $a\colon \mathbb{N} \to \mathbb{R}$ and $b\colon \mathbb{N} \to \mathbb{R}$ be given sequences. Define the sequence $c\colon \mathbb{N} \to \mathbb{R}$ by

$$c_n = a_n + b_n \quad \text{for every} \quad n \in \mathbb{N}.$$

Prove: If $\lim_{n\to+\infty} a_n = L$ and $\lim_{n\to+\infty} b_n = K$, then $\lim_{n\to+\infty} c_n = L + K$.

Problem 4. Consider the sequence $a\colon \mathbb{N} \to \mathbb{R}$ defined by

$$a_n = \sum_{k=n+1}^{2n} \frac{1}{k} = \frac{1}{n+1} + \cdots + \frac{1}{2n}, \qquad n \in \mathbb{N}.$$

Prove that this sequence converges. Finding the exact value of the limit is extra credit.

Problem 5. Prove that the sequence $s\colon \mathbb{N} \to \mathbb{R}$ defined by $s_n = (-1)^n$ for all $n \in \mathbb{N}$, does not converge.