Limit at $+\infty$

Motivation

Comment on Sam's post of today.

Recall the def of max.

$S \subseteq \mathbb{R}$, $S \neq \emptyset$ (nonempty set)

$u = \max S \iff$ ① $u \in S$
② $\forall x \in S \quad x \leq u$

Sam's Thm Let $A, B \subseteq \mathbb{R}$, A, B nonempty.
Assume that $\boxed{a = \max A}$ and $\boxed{b = \max B}$

Then $\max(A+B) = a+b$

Def. $A, B \subseteq \mathbb{R}$, nonempty.

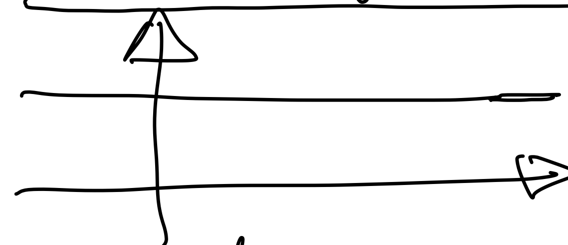
By def $\boxed{\underbrace{A+B}_{\text{new set}} = \{x+y : x \in A \wedge y \in B\}}$

Try to prove this — You can do it!

How does the definition of limit emerge?

(My answer!)

How would you define a **constant function**?



$f : A \to B$ codomain

domain

<u>Definition</u>

$\exists c \in B$ s.t. $\forall x \in A$ $f(x) = c$

<u>Practise negation</u>: $\forall c \in B$ $\exists x \in A$ s.t. $f(x) \neq c$

What does the phrase "eventually constant" mean?

We can do this only for real functions, $A, B \subseteq \mathbb{R}$.

Say $f : D \to \mathbb{R}$ where $D \subseteq \mathbb{R}$

$D \neq \emptyset$

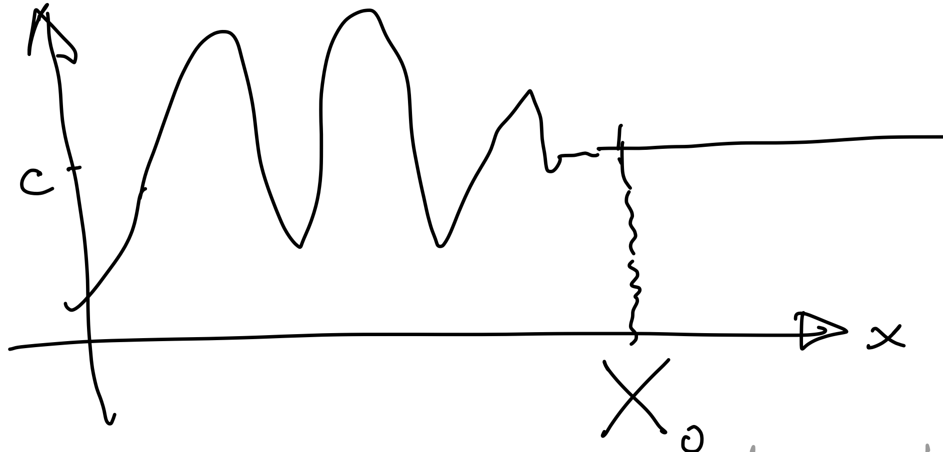
Def. eventually constant

$\exists c \in \mathbb{R}$ s.t.

$\exists X_0 \in \mathbb{R}$ s.t.

$\forall x > X_0$ we have

$f(x) = c$



In order for this def to make sense I must have $[X_0, +\infty) \subseteq D$

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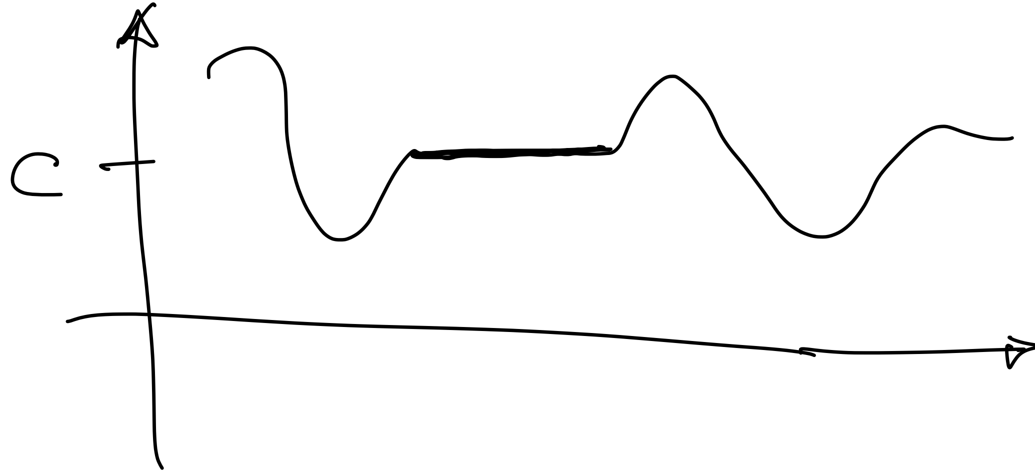
sgn, us, ... not many functions like this.

Let us play with made-up word: (informally)

"constantish" (meaning: like a constant) one way of understanding



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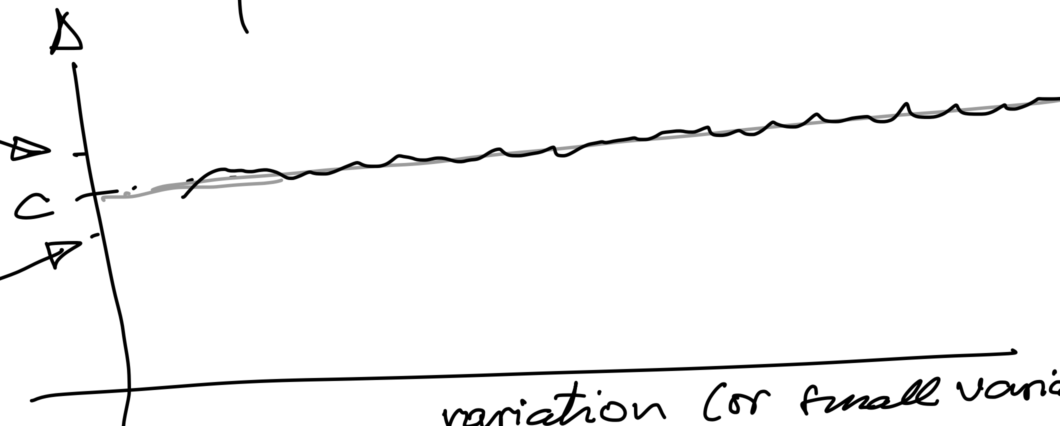

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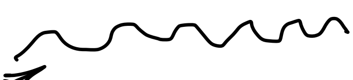
variation (or small variation)

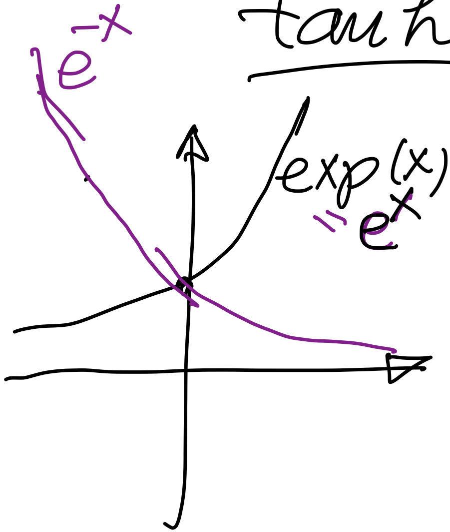
comment on chat

oscillation implies



would include this

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

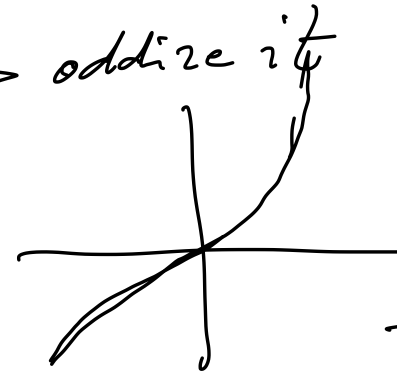


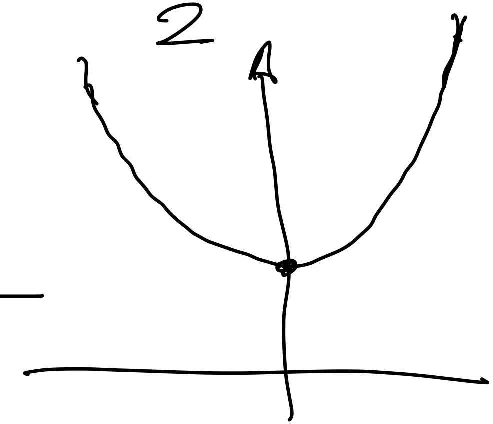
even → evenize it $\qquad \frac{e + e^{-x}}{2} = \cosh x$

vs

odd → oddize it

$$\frac{e^x - e^{-x}}{2}$$





$$\boxed{\tanh 8 = 1}, \quad \frac{e^8 - e^{-8}}{e^8 + e^{-8}}$$