More definitions of Limits, Squeeze Theorems

Definition Let $a, L \in \mathbb{R}$ and $D \subseteq \mathbb{R}$.

A function $f : D \to \mathbb{R}$ has the limit L as $x \to a$ if the following two conditions are satisfied:

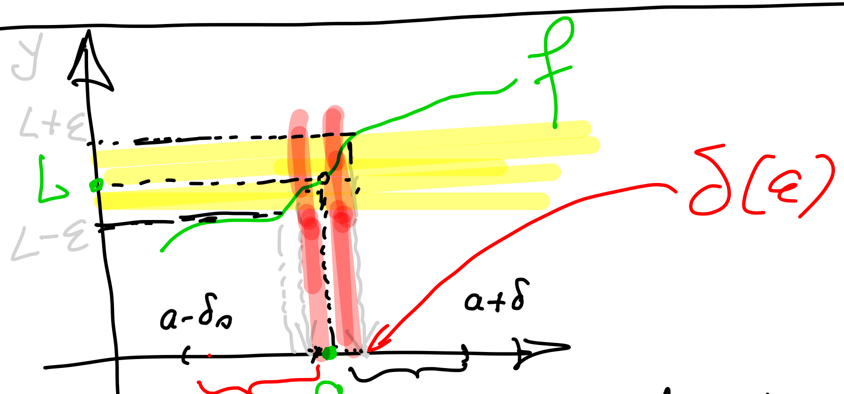
(I) $\exists\, \delta_0 > 0$ s.t. $(a-\delta_0, a) \cup (a, a+\delta_0) \subseteq D$,

left of a   right of a

(II) $\forall \varepsilon > 0\ \exists\, \delta(\varepsilon)$ such that $0 < \delta(\varepsilon) \le \delta_0$ and

$$0 < |x-a| < \delta(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$$

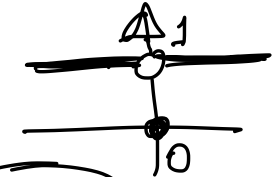
ε tied to f



δ_0 must be in the domain (I)

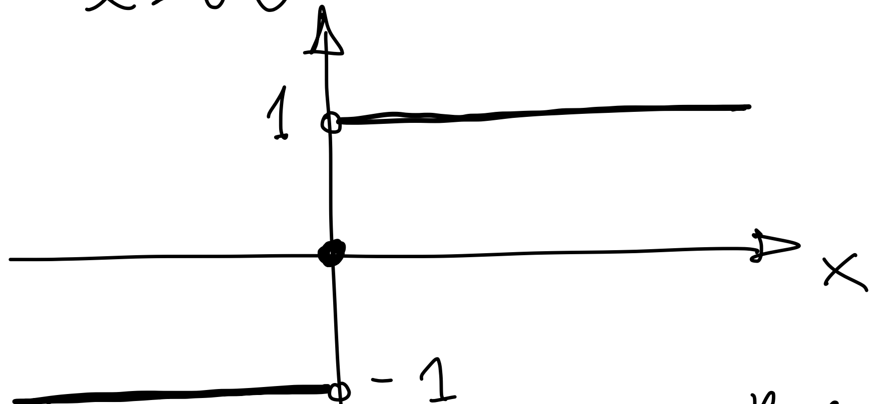
Simple examples

$g(x) = |\operatorname{sign}(x)|$



$\lim_{x\to 0} g(x) = 1$ $\quad a=0$

$\lim_{x\to 0} \operatorname{sign} x$ DNE



Def. Let $a, L \in \mathbb{R}$ and $D \subseteq \mathbb{R}$. A function $f : D \to \mathbb{R}$ has the limit L as x approaches a from the right if the following two cond. are satis:

right

(I) $\exists\, \delta_0 > 0$ s.t. $(a, a+\delta_0) \subseteq D$

(II) $\forall \varepsilon > 0 \;\; \exists\, \delta(\varepsilon)$ s.t. $0 < \delta(\varepsilon) \le \delta_0$ and

$0 < x-a < \delta(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$

right

The notation for this limit is

$$\lim_{x \searrow a} f(x) = L$$

right

$x \to a+$

$x \searrow a$

What are the changes for x approaches a from the left

$(a-\delta_0, a) \subseteq D$

$0 < a-x < \delta(\varepsilon) \Rightarrow \ldots$

In OUR main def. of limit the ABS measures the distance between two real numbers

To talk about <u>limit</u> of objects that are NOT real numbers we MUST have a concept of <u>distance</u> between those objects

Let A be a set of objects

B be a set of objects

We assume we have a concept of <u>distance</u> on A and on B

$\boxed{d_A(x_1, x_2)}$ distance

$x_1, x_2 \in A$

metric

$d_B(y_1, y_2)$

<u>Metric Spaces</u>

Must have prop. of distance:

1. $\forall x_1, x_2 \in A \quad d_A(x_1, x_2) \geq 0$
2. $\forall x_1, x_2 \in A \quad d_A(x_1, x_2) = 0 \Leftrightarrow x_1 = x_2$
3. $\forall x_1, x_2 \in A \quad d_A(x_1, x_2) = d_A(x_2, x_1)$
4. $\forall x_1, x_2, x_3 \in A$

$$d_A(x_1, x_3) \leq d_A(x_1, x_2) + d_A(x_2, x_3)$$

TRIANGLE INEQ. Must be true

Def. of Limit $a \in A$, $b \in B$

$\mathcal{D} \subseteq A$, $f : \mathcal{D} \to B$

the function f has the limit b

as $x \in A$ approaches $a \in A$

if the following two cond. are sat.

(I) leave for tomorrow | a is an accumulation point of $\mathcal{D}$ |

(II) $\forall \varepsilon > 0 \; \exists \delta(\varepsilon) > 0$ s.t.

$$0 < d_A(x, a) < \delta(\varepsilon) \Rightarrow d_B(f(x), b) < \varepsilon$$