

Consequences of the MCT

Definition of
Infinite Series,

① Definition of e

Then \mathbb{N} we define $S_n = \sum_{k=0}^n \frac{1}{k!}$.

→ we prove $\forall n \in \mathbb{N} \quad S_n \leq S_{n+1}$

→ we prove $\forall n \in \mathbb{N} \quad S_n \leq 3$

MTC $\Rightarrow \{S_n\}$ converges. Set

$$e = \lim_{n \rightarrow +\infty} S_n$$

e is irrational

(English)

$$\forall p \in \mathbb{Z} \quad \forall n \in \mathbb{N} \quad e \neq \frac{p}{q}$$

For a proof see
the short note
on the class
website.

$$(a+b)^2 = a^2 + 2ab + b^2$$

Here I explain the BINOMIAL THEOREM, needed in my short paper about e, see the website

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + ?a^3b + ?a^2b^2 + b^4$$

$$\underbrace{(a+b)^n}_{\text{binomial}} = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

↙ binomial coefficient

$$\frac{n!}{k!(n-k)!}$$

That is about it.

Now

$\sqrt{2}$.

$\sqrt{2}$

is irrational

$\forall p \in \mathbb{Z} \forall q \in \mathbb{N} \frac{p^2}{q^2} \neq 2$

Have you seen this proof?
Look for it, if not, review it
if yes.

But, PROVE

$\exists \alpha \in \mathbb{R}$ s.t. $\alpha^2 = 2$.

(We stated above $\forall \alpha \in \mathbb{Q} \quad \alpha^2 \neq 2$.)

We can now PROVE $\exists \alpha \in \mathbb{R}$ s.t. $\alpha^2 = 2$

We studied:

$x_1 = 2$, $x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$ then

We proved $\forall n \in \mathbb{N} \quad x_n > 0$
We proved $\forall n \in \mathbb{N} \quad x_{n+1} \leq x_n$

By MTC ~~EL~~ ER such that

$$\lim_{n \rightarrow \infty} x_n = L$$

We also proved $\forall (x_n)^2 \geq 2$

$\forall n \in \mathbb{N}$

Sorry if
You are very important

Sorry, I
forgot the
quantifier!

Now ALGEBRA of Limits:

$$a_n \rightarrow K \quad b_n \rightarrow L$$
$$\lim_{n \rightarrow +\infty} a_n \cdot b_n = K \cdot L$$

$$(x_n)^2 \geq 2 \quad \forall n \in \mathbb{N} \Rightarrow L^2 \geq 2$$

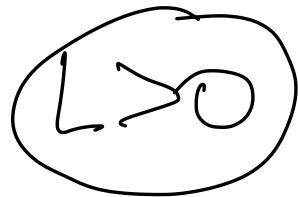
$$x_n > 0 \quad \forall n \in \mathbb{N} \Rightarrow L \geq 0$$

$$\Rightarrow L > 0$$

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n} \quad \forall n \in \mathbb{N}$$

$$\lim_{n \rightarrow \infty} \frac{x_n}{2} = \frac{L}{2}, \text{ Alg. of limits}$$

$$\lim_{n \rightarrow \infty} \frac{1}{x_n} = \frac{1}{L}$$



Alg. of Limits :

$$\lim_{n \rightarrow +\infty} \left(\frac{x_n}{2} + \frac{1}{x_n} \right) = \frac{L}{2} + \frac{1}{L}$$

Alg. of Limits :

$$\lim_{n \rightarrow +\infty} x_{n+1} = L$$

Since $x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n} \Rightarrow L = \frac{L}{2} + \frac{1}{L}$

$\forall n \in \mathbb{N}$

$$\frac{L}{2} = \frac{1}{L}$$

$$L^2 = 2$$

Here we proved

$$\exists L \in \mathbb{R} \text{ s.t. } L > 0 \text{ and } L^2 = 2$$

$$L > 0$$