MATH 302 Assignment 1 May 7, 2009

Name ______________________________

Problem 1. Let a, b, c, j, k be positive integers such that

$$a = cj, \qquad b = ck.$$

(a) Prove the implication: If $\operatorname{lcm}(j, k) = m$, then $\operatorname{lcm}(a, b) = cm$.

(b) Is the converse implication true? Justify your answer.

Problem 2. Let $k \in \mathbb{N}$. Let $t_k = \dfrac{k(k+1)}{2}$ be the k-th triangular number. Find the formula for $\gcd(t_k, t_{k+1})$ in terms of k. Prove that your formula is correct.

Problem 3. Let a and b be integers, not both zero. Prove that a and b are relatively prime if and only if there exists an integer c such that $a|c$ and $b|(c+1)$.

Problem 4. Let a and b be integers, not both zero. Let $d = \gcd(a, b)$. Prove that $\gcd(a^2, b^2) = d^2$. (Hint: First consider the special case of relatively prime integers a and b.)

Problem 5. Let a and b be positive integers. Prove that $(b^2)|(a^2)$ if and only if $b|a$.