MATH 304 Assignment 0 - Hints

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Proposition 1. *Prove that for all* $a \in (-1,1)$ *and all* $b \in (-1,1)$ *we have*

$$\frac{a+b}{1+ab} \in (-1,1). \tag{1}$$

Solution. Assume that $a \in (-1,1)$ and $b \in (-1,1)$. Then $|a| \in [0,1)$ and $|b| \in [0,1)$. Consequently, $|ab| \in [0,1)$. That is $ab \in (-1,1)$. Therefore $1+ab > 0$.

Since $a, b \in (-1,1)$, we have $1+a > 0$, $1-a > 0$, $1+b > 0$ and $1-b > 0$.

Since $1+a > 0$ and $1+b > 0$, we have $(1+a)(1+b) > 0$. Consequently, $1+a+b+ab > 0$ and hence

$$-1-ab < a+b.$$

As $1+ab > 0$, dividing both sides of the last inequality by $1+ab$ we get

$$-1 < \frac{a+b}{1+ab}. \tag{2}$$

Since $1-a > 0$ and $1-b > 0$, we have $(1-a)(1-b) > 0$. Consequently, $1-a-b+ab > 0$ and hence

$$a+b < 1+ab.$$

As $1+ab > 0$, dividing both sides of the last inequality by $1+ab$ we get

$$\frac{a+b}{1+ab} < 1. \tag{3}$$

Inequalities (2) and (3) prove inequality (1). □