

Winter 2014 Math 309 Topics for the final

Logic. Know:

- Truth table of the negation operator, conjunction, disjunction, exclusive disjunction, implication and biconditional.
- How to form the negation of an implication and contrapositive, converse, and inverse of an implication
- All different ways of saying p implies q
- How to prove tautologies, contradictions and logical equivalences using truth tables
- Logical equivalences, in particular distributive laws, De Morgan's laws and equivalences involving implications
- The meaning of the universal and the existential quantifier, and their negations
- How to work with nested quantifiers (how to state negations, how to recognize whether a statement is true or false and justify it, Exercises 26–33, 37, 38 and exercises on the web-site)
- The most important rules of inference: modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and the rules of inference for quantified statements
- Proofs from Section 1.5 related to odd/even integers, rational and irrational numbers (Example 14, Example 18, Example 19, Example 21, Example 24, and the corresponding Exercises 20–30)
- How to translate English sentences into logical propositions

Sets and Functions. Know

- The concept of a set, equality of sets, the concept of a subset, the empty set, cardinality of a finite set, the power set, Cartesian product
- Different set notations, set builder notation, use of ellipses, Venn diagrams,
- Set operations: intersection, union, set difference, complement, symmetric difference, and the corresponding set identities
- Proving set identities using a membership table
- The formal definition of a function (web-site) and the concepts of domain, codomain and range
- Definitions of a surjection, an injection and a bijection; how to recognize and prove whether a given function has these properties (Exercises 12, 13, 14, 17, 18)
- The concept of composition of functions and the inverse function and connections to the previous item (Exercises 25, 26, 27)
- Properties of the floor and the ceiling and how to use them to solve related exercises (Examples 24, 25, Exercises 48, 49, 65, 66)

Axioms and Propositions for \mathbb{Z} . Know (The numbers in this section relate to the document “Basic properties of the Integers”)

- The proof of Proposition 2.5. For every $a \in \mathbb{Z}$ we have $a \cdot 0 = 0 \cdot a = 0$.
- The proof of Corollary 3.7. $0 < 1$. (In class I gave a proof by contradiction.)
- Definitions 3.9 and 3.10 and Exercises 3.11 and 3.12.
- Section 4 up to Definition 4.5.
- Section 5, all except the proof of the Division Algorithm.

- Section 6.

Sequences, Induction and Recursion. Know

- Some common sequences, the basic properties of the summation notation
- The formulas for the sums of an arithmetic and a geometric progression
- The formal statement of the Principle of Mathematical Induction (and a proof from the notes “Basic properties ...”)
- How to do proofs involving both versions of the Mathematical Induction
- How recursive definitions work and proofs involving recursively defined functions

Counting. Know

- The basic counting principles and how to apply them to accurately count various sets
- How to apply the Pigeonhole principle in various situations
- How to use permutations and combinations to count various sets
- Basic identities involving permutations and combinations and how to prove them using algebraic and combinatorial methods
- Generalized permutations and combinations, how to count combinations and permutations with repetition
- How to use recursively defined sequences (recursive relations) for counting (Section 6.1)
- How to find closed form formula for a recursive sequence given by linear second-order homogeneous recursion (in book’s language: How to solve linear second-order homogeneous recurrence relations) (Section 6.2: Theorem 1, Examples 3, 4, Theorem 2, Example 5, Exercises 3, 11)
- How to find closed form solution of some recursive sequences given by linear second-order non-homogeneous recursions (in book’s language: How to solve linear second-order nonhomogeneous recurrence relations) (Section 6.2: Theorem 5, Examples 10, 11, Theorem 6, Example 12, 13, Exercises 23, 24, 25, 29, 32, 35)