Problems from Section 4.1

Problem 20

■ (a) How many integers in {1000,...,9999} are divisable by 9

We have 999 = 111*9. Thus the smallest integer in this set divisable by 9 is 112*9. The largest integer divisable by 9 in this set is 9999=1111*9. Thus there are

```
In[51]:= 1111 - 111

Out[51]= 1000
```

integers in the given set whichare divisable by 9

We can verify this in *Mathematica* by the following commands

```
In[52]:= IntegerQ[#/9] &[7866]

Out[52]= True

In[53]:= Length[Select[Range[1000, 9999], IntegerQ[#/9] &]]

Out[53]= 1000
```

■ (b) How many integers in {1000,...,9999} are even

1000 = 500*2 is even and 4999*2 = 9998 is even. Thus, there are

```
In[54]:= 4999 - 500 + 1

Out[54]= 4500
```

even integers in this set.

Verification in *Mathematica*

```
In[55]:= Length[Select[Range[1000, 9999], EvenQ[#] &]]

Out[55]= 4500
```

■ (c) How many integers in {1000,...,9999} have distinct digits

We calculate this by using the product rule. There are 9 options, that is {1, 2, 3, 4, 5, 6, 7, 8, 9}, to choose the first digit; say this is d_1; there are 9 options, that is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\backslash\{d_1\}$, to choose the second digit; say this is d_2; there are 8 options, that is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\backslash\{d_1, d_2\}$, to choose the third digit; say this is d_3; and finally, there are 7 options, that is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\backslash\{d_1, d_2, d_3\}$, to choose the fourth digit.

```
In[56]:= 9 9 8 7

Out[56]= 4536
```

Verification in *Mathematica*

The command

```
In[57]:= Length[Union[IntegerDigits[#]]] &[3456]

Out[57]= 4
```

tells us how many DISTINCT digits there are in an integer. The command

```
In[58]:= Length[Union[IntegerDigits[#]]] == 4 &[3456]

Out[58]= True
```

tells if the number of DISTINCT digits is equal to 4

Finally this is *Mathematica*'s answer to (c)

```
In[59]:= Length[Select[Range[1000, 9999], Length[Union[IntegerDigits[#]]] == 4 &]]

Out[59]= 4536
```

■ (d) How many integers in {1000,...,9999} are not divisable by 3

999=333*3 and 9999=3333*3, so there are 3000 integers divisable by 3. Since the total is 9000 integers, there are 6000 integers not divisable by 3

```
In[60]:= Length[Select[Range[1000, 9999], Not[IntegerQ[#/3]] &]]

Out[60]= 6000
```

■ (e), (f), (g), (h)

The relevant counts are:

the number of integers divisable by 5

In[61]:= **div5 = Floor[$\frac{9999}{5}$] - Floor[$\frac{999}{5}$]**

Out[61]= 1800

the number of integers divisable by 7

In[62]:= **div7 = Floor[$\frac{9999}{7}$] - Floor[$\frac{999}{7}$]**

Out[62]= 1286

the number of integers divisable by both 5 and 7

In[63]:= **div35 = Floor[$\frac{9999}{35}$] - Floor[$\frac{999}{35}$]**

Out[63]= 257

So, the answer to (e), the number of integers divisable by 5 or 7 is

In[64]:= **div5 + div7 - div35**

Out[64]= 2829

Mathematica verification

In[65]:= **Length[Select[Range[1000, 9999], Or[IntegerQ[$\frac{\#}{7}$], IntegerQ[$\frac{\#}{5}$]] &]]**

Out[65]= 2829

The answer to (f),the number of integers not divisable by either 5 or 7 is

In[66]:= **9000 - (div5 + div7 - div35)**

Out[66]= 6171

Mathematica verification

In[67]:= **Length[Select[Range[1000, 9999], Not[Or[IntegerQ[$\frac{\#}{7}$], IntegerQ[$\frac{\#}{5}$]]] &]]**

Out[67]= 6171

The answer to (g), the number of integers divisable by 5 but not divisable by 7 is

In[68]:= **div5 - div35**

Out[68]= 1543

Mathematica verification

In[69]:=
```
Length[Select[Range[1000, 9999], And[Not[IntegerQ[#/7]], IntegerQ[#/5]] &]]
```

Out[69]= 1543

The answer to (h), the number of integers divisable by 5 and divisable by 7 is

In[70]:=
```
div35
```

Out[70]= 257

Mathematica verification

In[71]:=
```
Length[Select[Range[1000, 9999], And[IntegerQ[#/7], IntegerQ[#/5]] &]]
```

Out[71]= 257

To verify problems involving strings we need

In[72]:=
```
<< DiscreteMath`Combinatorica`
```

In[73]:=
```
Strings[{0, 1}, 4]
```

Out[73]= {{0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 0, 1, 1}, {0, 1, 0, 0}, {0, 1, 0, 1}, {0, 1, 1, 0}, {0, 1, 1, 1}, {1, 0, 0, 0}, {1, 0, 0, 1}, {1, 0, 1, 0}, {1, 0, 1, 1}, {1, 1, 0, 0}, {1, 1, 0, 1}, {1, 1, 1, 0}, {1, 1, 1, 1}}

Problem 40

How many bit strings of length 7 start with 00 or end with 111?

There are 32 strings which start with 00 and there are 16 strings which end with 111. There are 4 strings that start with 00 and end with 111. Thus the answer is by inclusion-exclusion rule

In[74]:=
```
32 + 16 - 4
```

Out[74]= 44

Mathematica verification

The following command will tell me which is the first bit in a bitstring

In[75]:=
```
#[[1]] &[{1, 0, 0, 1, 0, 1, 1}]
```

Out[75]= 1

The following command will tell me what problem is asking for

```
In[76]:= Or[And[#[[1]] == 0, #[[2]] == 0], And[#[[7]] == 1, #[[6]] == 1, #[[5]] == 1]] &[{1, 0, 1, 0, 0, 1, 1}]

Out[76]= False

In[77]:= Or[And[#[[1]] == 0, #[[2]] == 0], And[#[[7]] == 1, #[[6]] == 1, #[[5]] == 1]] &[{0, 0, 1, 0, 0, 1, 1}]

Out[77]= True

In[78]:= Length[Select[Strings[{0, 1}, 7],
           Or[And[#[[1]] == 0, #[[2]] == 0], And[#[[7]] == 1, #[[6]] == 1, #[[5]] == 1]] &]]

Out[78]= 44
```

Problem 42

How many bit strings of length 10 contain at least 5 consecutive 0s or at least 5 consecutive 1s?

We will first count the bitstrings with at least 5 consecutive 0s.

The "at least 5 consecutive 0s" can start at the following positions 1, 2, 3, 4, 5, 6

There are $2^5 = 32$ bit srings which start with 00000***** (type 1)

There are $2^4 = 16$ bit srings which start with 100000**** (type 2)

There are $2^4 = 16$ bit srings which start with *100000*** (type 3)

There are $2^4 = 16$ bit srings which start with **100000** (type 4)

There are $2^4 = 16$ bit srings which start with ***100000* (type 5)

There are $2^4 = 16$ bit srings which start with ****100000 (type 6)

There are no bitstrings that belong to two different types. (You conclude this by looking at types j and k, j < k. Type k has 1 at position k-1 while type j has 0 at the position k-1.

Thus there are

```
In[79]:= 32 + 5 * 16

Out[79]= 112
```

bit strings with at least 5 consecutive 0s

Also, there are 112 bit strings with at least 5 consecutive 1s.

There are 2 bit strings which are in both sets: 0000011111 and 1111100000.

By the inclusion-exclusion principle the answer is

```
In[80]:= 2*112 - 2

Out[80]= 222
```

To verify this in *Mathematica* is a little bit more complicated.

The following command will collect the identical consecutive bits in separate lists

```
In[81]:= Split[{1, 0, 1, 0, 0, 1, 0, 0, 0, 0}]

Out[81]= {{1}, {0}, {1}, {0, 0}, {1}, {0, 0, 0, 0}}
```

The following command will count how many consecutive bits there are

```
In[82]:= Length[#] & /@ Split[{1, 0, 1, 0, 0, 1, 0, 0, 0, 0}]

Out[82]= {1, 1, 1, 2, 1, 4}
```

The following command will find the maximum number of consecutive bits

```
In[83]:= Max[Length[#] & /@ Split[{1, 0, 1, 0, 0, 1, 0, 0, 0, 0}]]

Out[83]= 4
```

And finally we will ask if that max number is ≥5.

```
In[84]:= Max[Length[#] & /@ Split[{1, 0, 1, 0, 0, 1, 0, 0, 0, 0}]] ≥ 5

Out[84]= False
```

Make this into a function of a bit string

```
In[85]:= Max[Length[#] & /@ Split[#]] ≥ 5 &[{1, 0, 1, 0, 0, 1, 0, 0, 0, 0}]

Out[85]= False

In[86]:= Length[Select[Strings[{0, 1}, 10], Max[Length[#] & /@ Split[#]] ≥ 5 &]]

Out[86]= 222
```