Winter 2019 Math 309 Topics for Final Exam

Logic. Know:

- Truth table of the negation operator, conjunction, disjunction, exclusive disjunction, implication and biconditional.
- How to form the negation of an implication and contrapositive, converse, and inverse of an implication
- All different ways of saying p implies q
- How to prove tautologies, contradictions and logical equivalences using truth tables
- Logical equivalences, in particular distributive laws, De Morgan's laws and equivalences involving implications
- The meaning of the universal and the existential quantifier, and their negations
- How to work with nested quantifiers (how to state negations, how to recognize whether a statement is true or false and justify it, Exercises 26–33, 37, 38 and exercises on the web-site posted on January 17, 2019)
- The most important rules of inference: modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and the rules of inference for quantified statements
- Proofs from Section 1.5 related to odd/even integers, rational and irrational numbers (Example 14, Example 18, Example 19, Example 21, Example 24, and the corresponding Exercises 20–30)
- A direct proof that $\sqrt{2}$ is irrational posted on January 18, 2019.
- How to translate English sentences into logical propositions

Sets and Functions. Know

- The concept of a set, equality of sets, the concept of a subset, the empty set, cardinality of a finite set, the power set, Cartesian product
- Different set notations, set builder notation, use of ellipses, Venn diagrams
- Set operations: intersection, union, set difference, complement, symmetric difference, and the corresponding set identities
- Proving set identities using a membership table
- The formal definition of a function (web-site) and the concepts of domain, codomain and range
- Definitions of a surjection, an injection and a bijection; how to recognize and prove whether a given function has these properties (Exercises 12, 13, 14, 17, 18)
- The concept of composition of functions and the inverse function and connections to the previous item
- Properties of the floor and the ceiling and how to use them to solve related exercises (Examples 24, 25, Exercises 48, 49, 65, 66)

Axioms and Propositions for $\mathbb{Z}$. Know (The numbers in this section relate to the document "Basic properties of the Integers" posted on the class website)

- Section 2, Propositions 2.1, 2.2, 2.7
- Section 3, Proposition 3.2, Corollaries: 3.5, 3.6, Definitions 3.12 and 3.13, Exercises 3.14 and 3.15
- Section 4, Several equivalent formulations of Axiom WO, Definition 4.1, Propositions 4.2 and 4.3
- Section 5, Theorem 5.1

Sequences, Induction and Recursion. Know

- ➢ Some common sequences, the basic properties of the summation notation
- ➢ The formulas for the sums of an arithmetic progression and a geometric progression with proofs
- ➢ The definition of a countable set and how to prove that the set $\mathbb{Z}$ is countable.
- ➢ The concept of cardinality for sets and how to prove that $\mathcal{P}(\mathbb{Z}^+)$ is not countable.
- ➢ The formal statement of the Principle of Mathematical Induction (and a proof from the notes "Basic properties of the Integers")
- ➢ How to do proofs involving both versions of the Mathematical Induction
- ➢ How recursive definitions work and proofs involving recursively defined functions

Counting. Know

- ➢ The basic counting principles and how to apply them to accurately count various sets
- ➢ How to use permutations and combinations to count various sets
- ➢ Basic identities involving permutations and combinations and how to prove them using algebraic and combinatorial methods
- ➢ Generalized permutations and combinations, how to count combinations and permutations with repetition
- ➢ How to use recursively defined sequences (recursive relations) for counting (Section 6.1, Example 5, Example 6, Exercises 22, 23, 24, 25, 26, 27)