Section 2.1 version September 29, 2011 at 09:45

Assigned problems: 3, 4, 5, 6, 7, 10, 13, 14, 15, 17,18, 20, (21, 22, 24, 31, 32).

Selected solutions:

3. The differential equation is

$$y' = -ty.$$

The proposed general solution is

$$y(t) = Ce^{-(1/2)t^2}.$$

The derivative of the proposed general solution is

$$y'(t) = Ce^{-(1/2)t^2}(-t) = -t\left(Ce^{-(1/2)t^2}\right) = -ty(t).$$

Hence, the proposed general solution satisfies the differential equation. Choosing $C = -3, -2, -1, 0, 1, 2, 3$ we get Figure 1

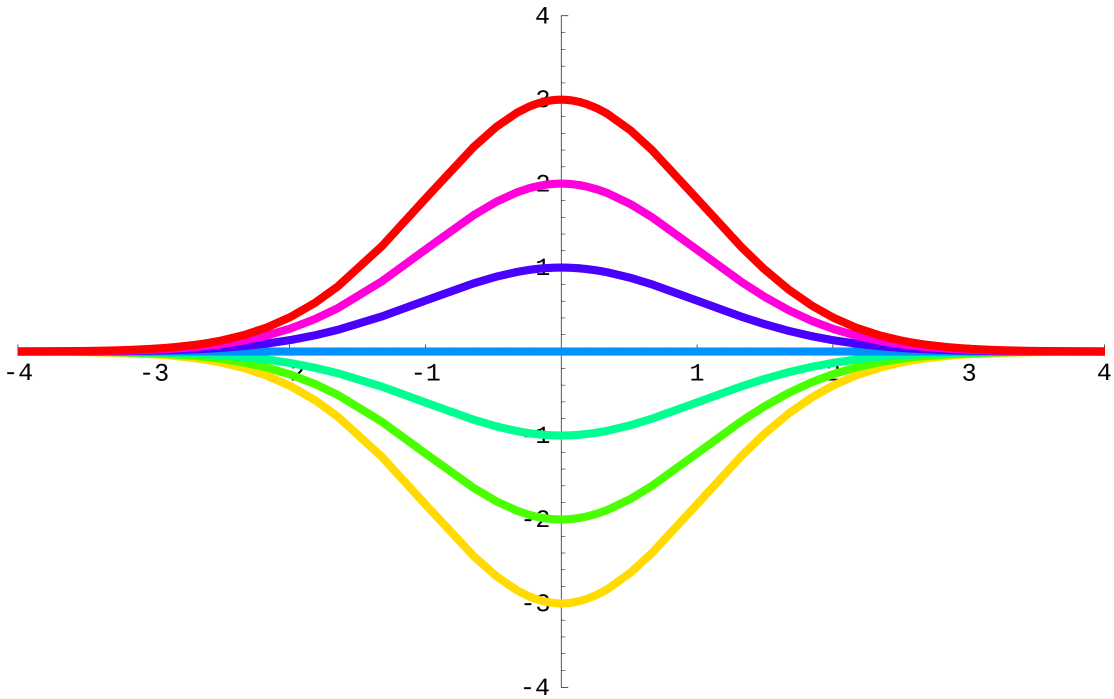

Figure 1: Problem 3

4. The differential equation is

$$y' + y = 2t.$$

The proposed general solution is

$$y(t) = 2t - 2 + Ce^{-t}.$$

The derivative of the proposed general solution is

$$y'(t) = 2 - Ce^{-t}.$$

Thus

$$y'(t) + y(t) = 2 - Ce^{-t} + 2t - 2 + Ce^{-t} = 2t.$$

Hence, the proposed general solution satisfies the differential equation. Choosing $C = -3, -2, -1, 0, 1, 2, 3$ we get we get Figure 2

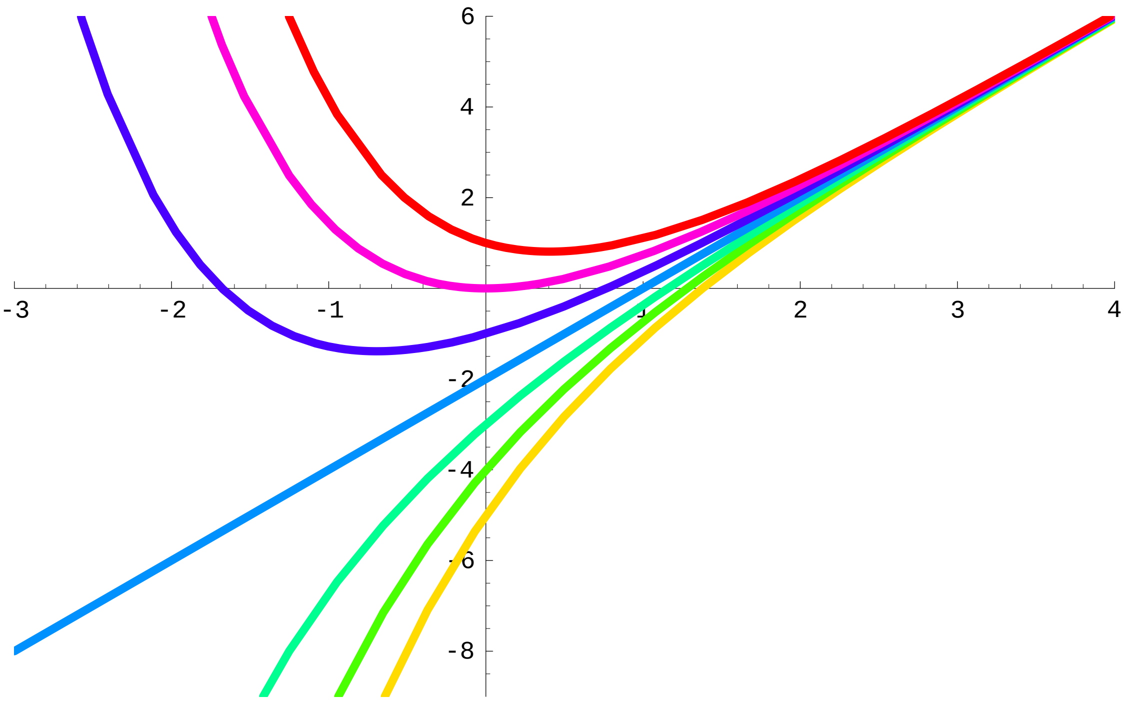

Figure 2: Problem 4

5. The differential equation is

$$y' + \frac{1}{2}y = 2\cos(t).$$

The proposed general solution is

$$y(t) = \frac{4}{5}\cos(t) + \frac{8}{5}\sin(t) + Ce^{-t/2}$$

The derivative of the proposed general solution is

$$y'(t) = -\frac{4}{5}\sin(t) + \frac{8}{5}\cos(t) - \frac{1}{2}Ce^{-t/2}.$$

Thus

$$y'(t) + \frac{1}{2}y(t) = -\frac{4}{5}\sin(t) + \frac{8}{5}\cos(t) - \frac{1}{2}Ce^{-t/2} + \frac{2}{5}\cos(t) + \frac{4}{5}\sin(t) + \frac{1}{2}Ce^{-t/2} = 2\cos(t).$$

Hence, the proposed general solution satisfies the differential equation. Choosing $C = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ we get we get Figure 3

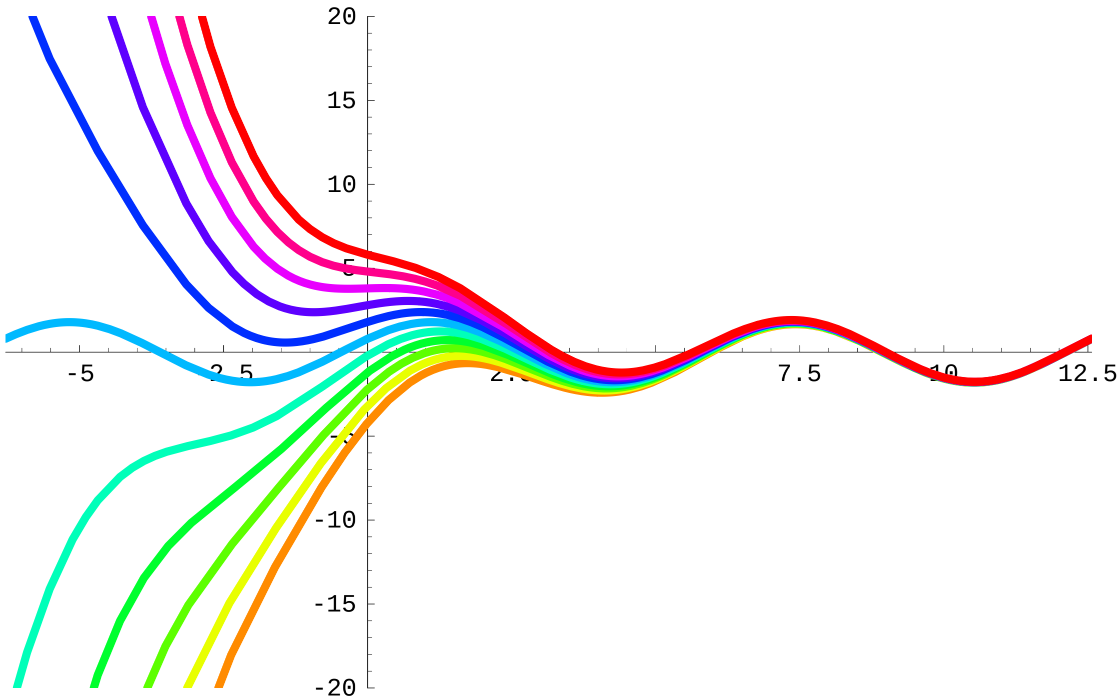

Figure 3: Problem 5

6. The differential equation is

$$y' = y(4-y).$$

The proposed general solution is

$$y(t) = \frac{4}{1+Ce^{-4t}}$$

The derivative of the proposed general solution is

$$y'(t) = -\frac{4}{(1+Ce^{-4t})^2}C(-4)e^{-4t} = \frac{16Ce^{-4t}}{(1+Ce^{-4t})^2}.$$

Now calculate

$$y(t)\big(4-y(t)\big) = \frac{4}{1+Ce^{-4t}}\left(4-\frac{4}{1+Ce^{-4t}}\right) = \frac{4\big(4+4Ce^{-4t}-4\big)}{(1+Ce^{-4t})^2} = \frac{16Ce^{-4t}}{(1+Ce^{-4t})^2} = y'(t).$$

Hence, the proposed general solution satisfies the differential equation. Choosing $C = 1, 2, 3, 4, 5$ we get we get Figure 4

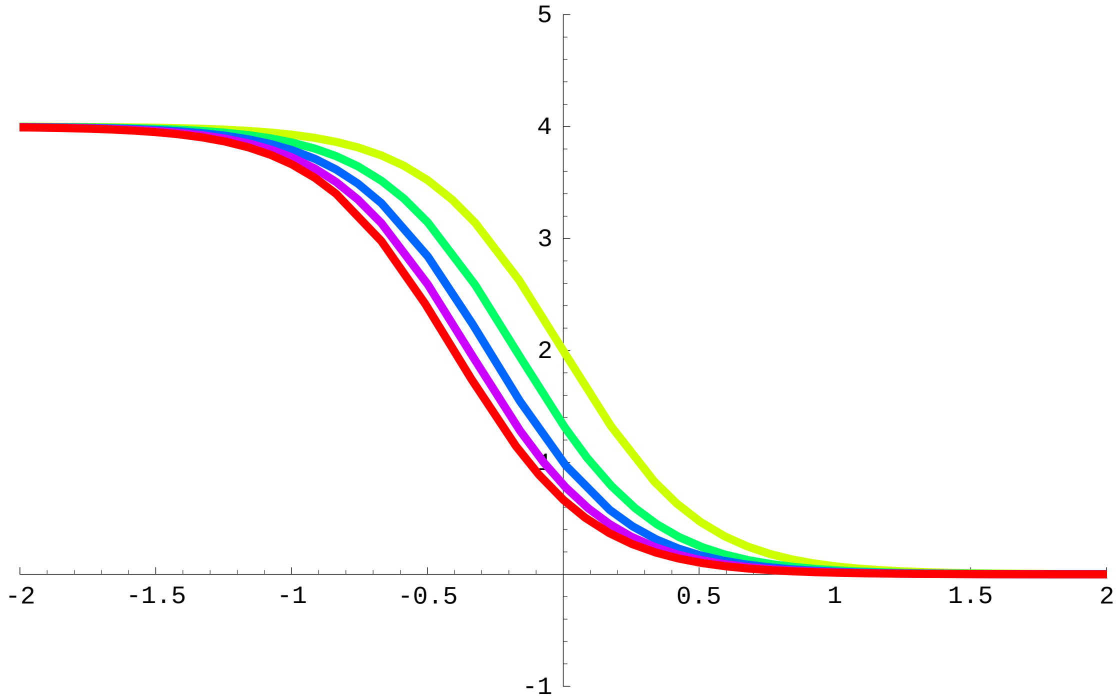

Figure 4: Problem 6

7. The constant function $y(t) = 0$ is a solution of the differential equation in Problem 6. Why? The derivative of this function is 0. Now evaluate the right-hand side substituting $y(t) = 0$:

$$y(t)\big(4 - y(t)\big) = 0(4 - 0) = 0 \cdot 4 = 0 = y'(t).$$

Thus the constant function 0 is a solution. But there is no value of C such that

$$\frac{4}{1 + Ce^{-4t}} = 0.$$

For a fraction to be 0 the numerator must be 0. In this case numerator is 4 and, as we very well know, $4 \neq 0$.

10. The proposed solution is

$$y(t) = \frac{3}{6t - 11} = 3(6t - 11)^{-1}.$$

First observe that

$$y(2) = \frac{3}{62 - 11} = 3.$$

Hence, this function satisfies the initial condition. Now calculate the derivative

$$y'(t) = -3(6t - 11)^{-2}\, 6 = -18(6t - 11)^{-2} = -2\big(3^2(6t - 11)^{-2}\big) = -2y(t)^2.$$

Thus, the given function is really a solution of the given initial value problem.
The function $y(t)$ is not defined at $t = 11/6$. Since $11/6 < 2$, the interval of existence of the solution is $t > 11/6$, that is $(11/6, +\infty)$. This is illustrated at Figure 5.

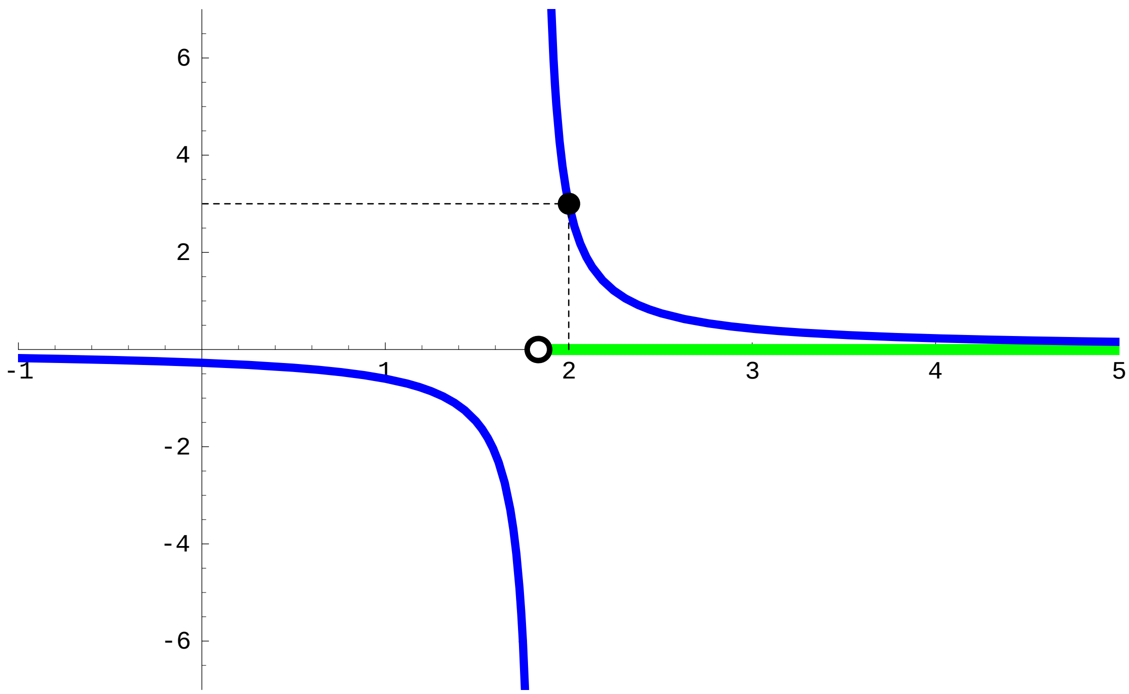

Figure 5: Problem 10

13. The proposed solution is

$$y(t) = \frac{1}{3}t^2 + \frac{C}{t}.$$

You should verify that this is really the general solution of the given differential equation. We will solve the given initial value problem:

$$y(1) = \frac{1}{3} + \frac{C}{1} = 2.$$

Solving for C we get $C = 5/3$. Thus the solution is

$$y(t) = \frac{1}{3}t^2 + \frac{5}{3t}.$$

The function $y(t)$ is not defined at $t = 0$. Since $0 < 1$, the interval of existence of the solution is $t > 0$, that is $(0, +\infty)$. This is illustrated at Figure 6.

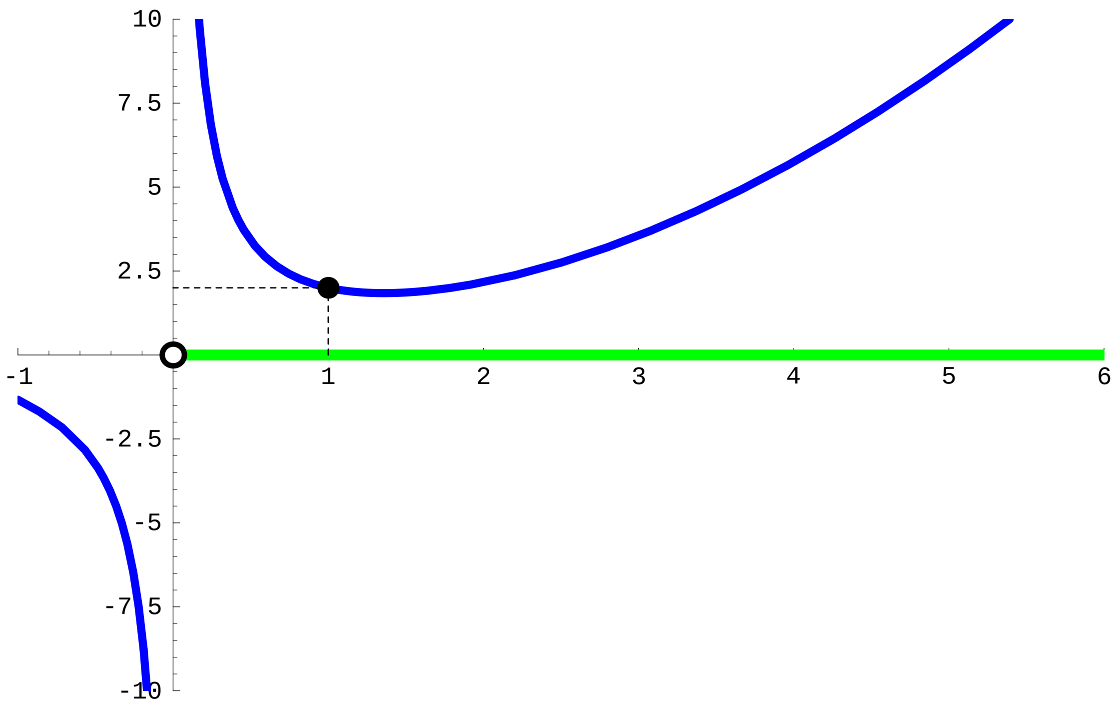

Figure 6: Problem 13

14. The proposed solution is

$$y(t) = e^{-t}\left(t + \frac{C}{t}\right).$$

You should verify that this is really the general solution of the given differential equation. We will solve the given initial value problem:

$$y(1) = e^{-1}\left(1 + \frac{C}{1}\right) = \frac{1}{e}.$$

Since $e^{-1} = 1/e$, solving for C yields $C = 0$. Thus the solution is

$$y(t) = te^{-t}.$$

The function $y(t)$ is defined for all real numbers. The interval of existence of the solution is $(-\infty, +\infty)$. This is illustrated at Figure 7.

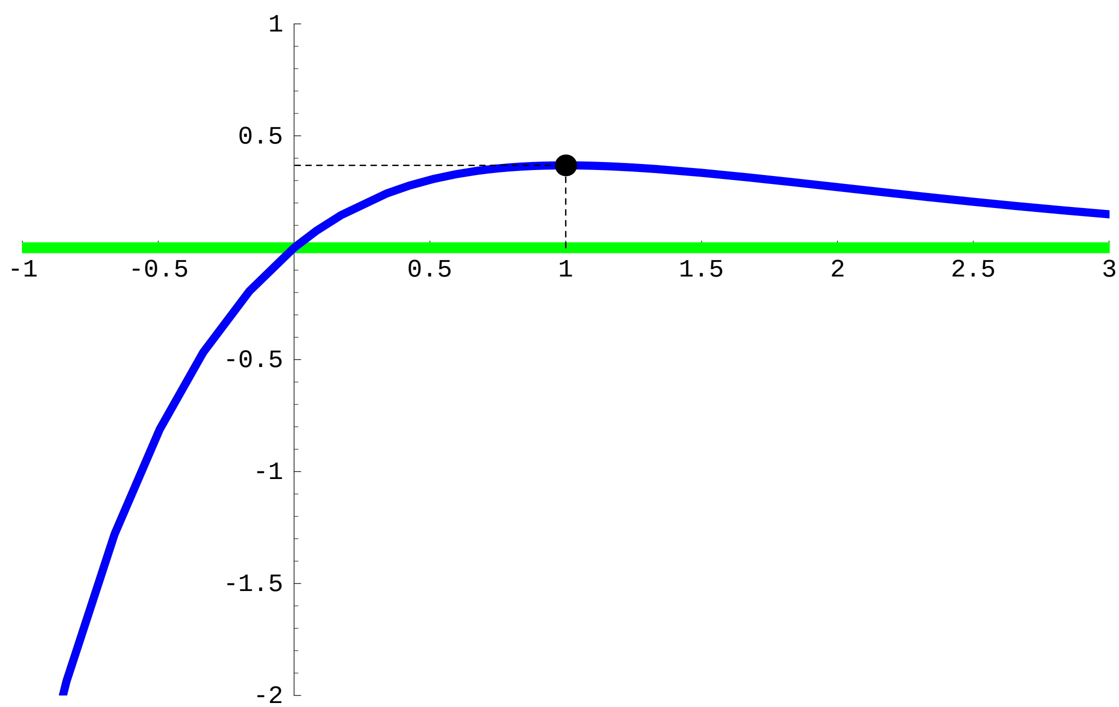

Figure 7: Problem 14

15. The proposed solution is

$$y(t) = \frac{2}{-1 + Ce^{-2t}}.$$

You should verify that this is really the general solution of the given differential equation. We will solve the given initial value problem:

$$y(0) = \frac{2}{-1 + Ce^{0}} = -3.$$

Simplifying we get $2 = 3 - 3C$, that is $C = 1/3$. Thus the solution is

$$y(t) = \frac{6}{-3 + e^{-2t}}.$$

The function $y(t)$ is not defined for $t = -(\ln 3)/2$. Since $-(\ln 3)/2 < 0$, the interval of existence of the solution is $t > -(\ln 3)/2$, that is $\big(-(\ln 3)/2, +\infty\big)$. This is illustrated at Figure 8.

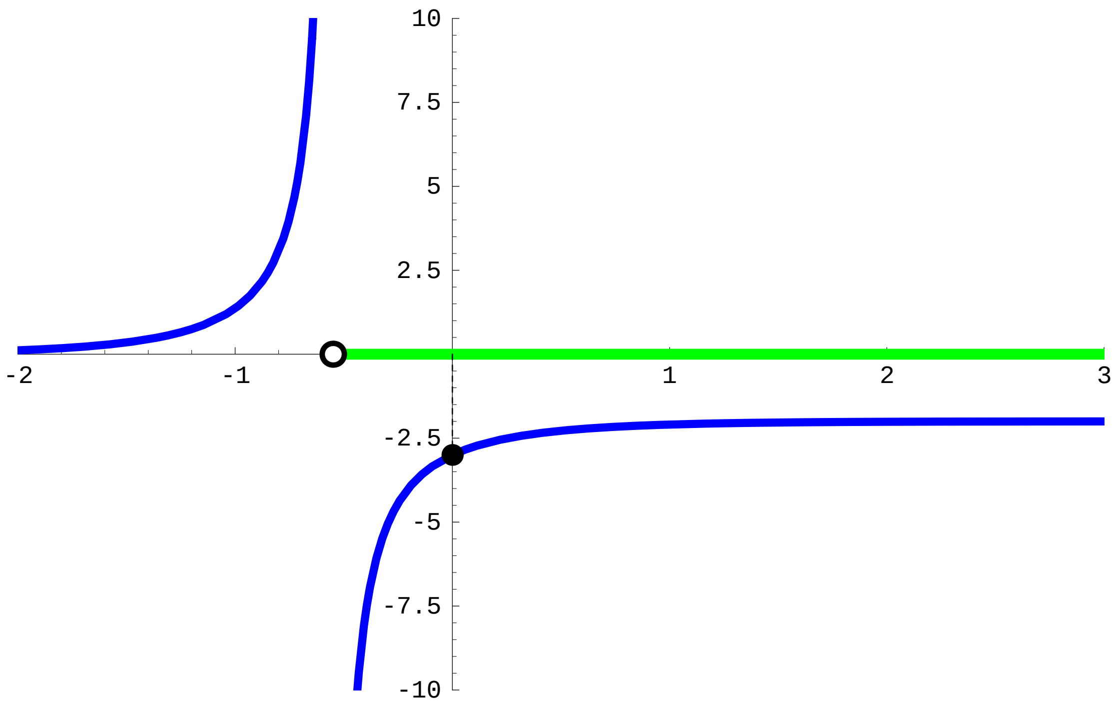

Figure 8: Problem 15