Section 5.4 version December 1, 2011 at 22:04

Exercises

17. We need to solve the initial value problem

$$y''(t) + y'(t) = t\,e^{-t}, \quad y(0) = -2, \quad y'(0) = 0.$$

Applying the Laplace transform to both sides of the equation we get

$$s^2\,Y(s) + 2s + s\,Y(s) + 2 = \frac{1}{(s+1)^2}.$$

Solving for $Y(s)$ yields

$$Y(s) = \frac{1}{s(s+1)(s+1)^2} - \frac{2(s+1)}{s(s+1)} = \frac{1}{s(s+1)^3} - \frac{2}{s}$$

Now the only problem is to find A, B, C, D such that

$$\frac{1}{s(s+1)^3} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}.$$

Writing the rational expressions on the right-hand side and equating the resulting numerator to 1 we get the equations for A, B, C, D:

$$A = 1, \quad 3A + B + C + D = 0, \quad 3A + 2B + C = 0, \quad A + B = 0.$$

Clearly the solution is $A = 1, B = -1, C = -1, D = -1$. Hence,

$$\frac{1}{s(s+1)^3} = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} - \frac{1}{(s+1)^3}.$$

Thus,

$$Y(s) = \frac{1}{s(s+1)^3} - \frac{2}{s} = -\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} - \frac{1}{2}\,\frac{2!}{(s+1)^3}.$$

All functions in the last expression for $Y(s)$ above are in the table of Laplace transforms. (Look at both: the special and the general rules.) Therefore the inverse Laplace transform of $Y(s)$ is (see Figure 1)

$$y(t) = -1 - e^{-t} - t\,e^{-t} - \frac{1}{2}\,t^2\,e^{-t}.$$

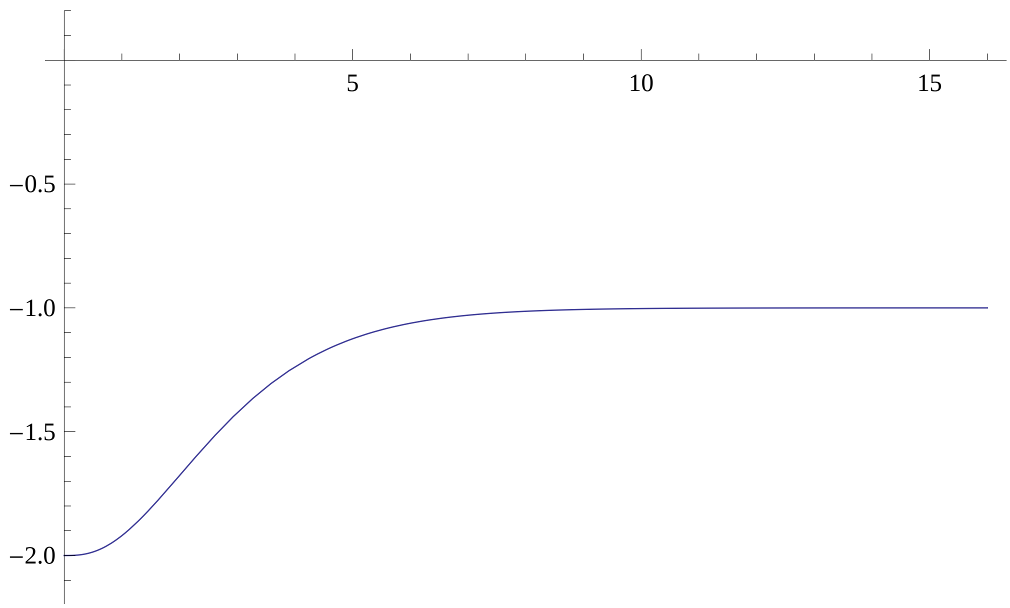

Figure 1: Problem 17