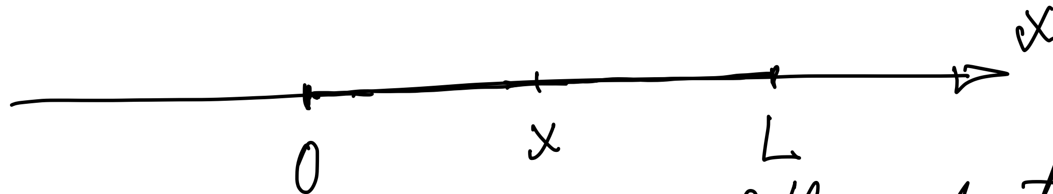
Heat Equation,
Boundary Conditions



$u(x,t)$ temperature of the rod at point x and time t

$c(x)$ specific heat

$\rho(x)$ mass density

$K_0(x)$ Fourier's constant

Heat Sources

$$c(x)\,\rho(x)\,\frac{\partial u}{\partial t}(x,t) = \frac{\partial}{\partial x}\left(K_0(x)\,\frac{\partial u}{\partial x}(x,t)\right)+Q(x,t)$$

Initial Condition $u(x,0) = f(x)$ $\quad 0 \le x \le L$

Boundary Conditions: $u(0,t) = a(t)$ $\quad u(L,t) = b(t)$

BCs ① temperature prescribed at the endpoints 0 & L

$$u(0,t)=a(t) \qquad u(L,t)=b(t)$$

boundary point

BP

$$\boxed{u(0,t)=0, \quad u(L,t)=0}$$

Diriclet's bc-s

special

② heat flux is prescribed at the endpoints 0 & L

$$\phi(x,t) = -K_0(x)\frac{\partial u}{\partial x}(x,t)$$

$$-K_0(0)\frac{\partial u}{\partial x}(0,t)=\alpha(t) \qquad -K_0(L)\frac{\partial u}{\partial x}(L,t)=\beta(t)$$

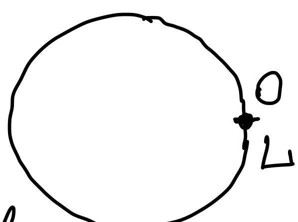
Special case: insulated boundary points
(the most import.)

$$\frac{\partial u}{\partial x}(0,t)=0 \qquad \frac{\partial u}{\partial x}(L,t)=0$$

Neuman's conditions

$$\forall t \geq 0$$

③



heated ring

$$u(0,t)=u(L,t)$$

only one BC

$$\frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(L,t)$$

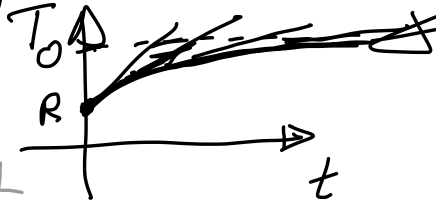
second BC

periodic BCs

assume

$$\boxed{\begin{aligned}\rho(0)&=\rho(L)\\ c(0)&=c(L)\\ K_0(0)&=K_0(L)\end{aligned}}$$

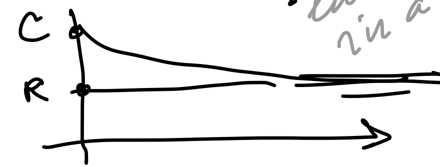
4. These BCs come from Newton's Law of cooling (331)



potato in an oven

$$\frac{dT}{dt} = h\left(b - T(t)\right)$$

$h > 0$



hot coffee in a room

$$-K_0(L)\frac{\partial u}{\partial x}(L,t) = h\left(u(L,t) - \delta(t)\right)$$

environment temperature

$$-K_0(0)\frac{\partial u}{\partial x}(0,t) = -h\left(u(0,t) - \gamma(t)\right)$$