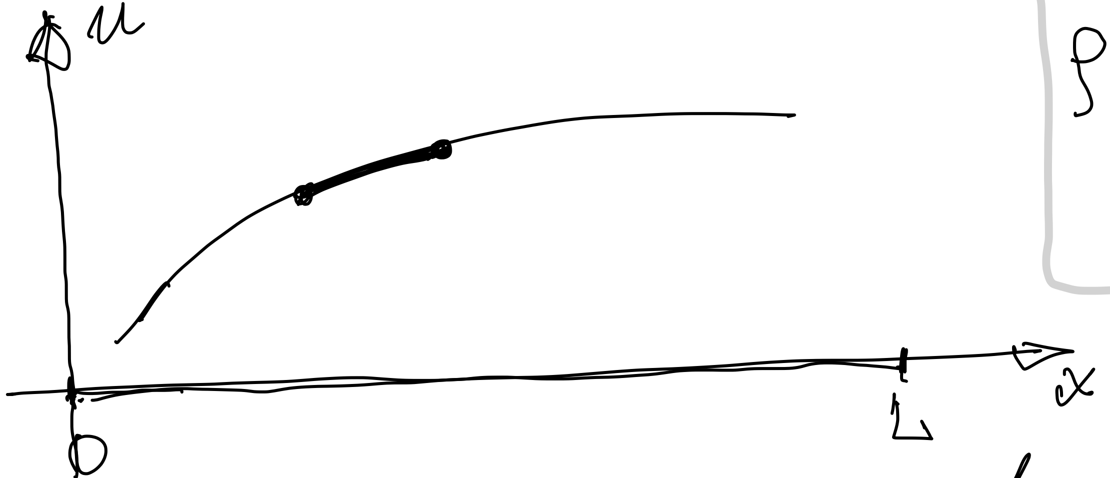
Boundary Conditions for the Vibrating String Equation



$$\rho(x)\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial}{\partial x}\left(T(x)\,\frac{\partial u}{\partial x}(x,t)\right)$$

$u(x,t)$ displacement at position x at time t as it vibrates

$$\frac{\partial^2 u}{\partial t^2}(x,t) = \mathcal{H}\,\frac{\partial^2 u}{\partial x^2}(x,t)$$

$\mathcal{H} \approx \frac{T_0}{\rho_0}$ string constant

with ρ & T constant

Next: Boundary Conditions and $\quad$ B.C.s

Initial Conditions $\quad$ I.C.s

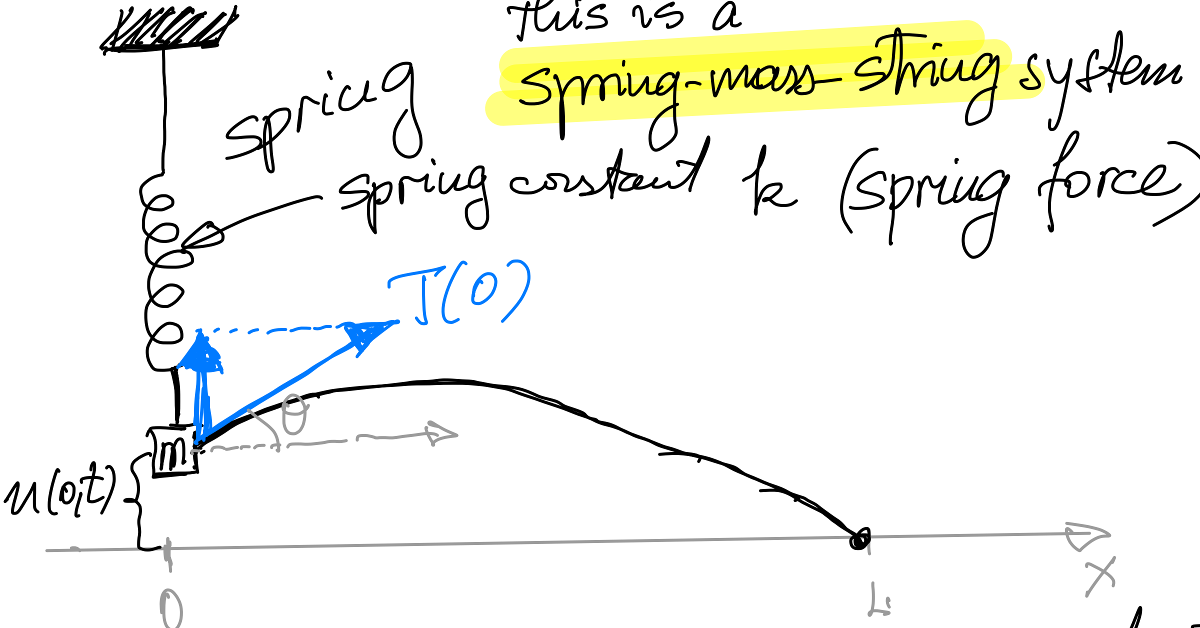
B.C.s

$$\frac{\partial^2 u}{\partial t^2} = \varkappa \frac{\partial^2 u}{\partial x^2}$$

$$u(0,t) = 0 \quad \forall t \in \mathbb{R}_+$$
$$u(L,t) = 0$$

Dirichlet BCs

this is a spring-mass-string system



Did you do Spring-mass systems in 331

We assume that the spring is at the equilibrium at x-axis. In this picture the spring is compressed, so it exerts a downward force on the mass

The spring force is proportional to the compression

$$F = \underbrace{ma}$$

$$m\frac{\partial^2 u}{\partial t^2}(0,t) = \underbrace{-k\cdot u(0,t)}_{\text{the spring force}} + T(0)\sin(\theta(0))$$

θ small

$\tan(\theta(0))$

$= \frac{\partial u}{\partial x}(0,t)$

$$\boxed{m\frac{\partial^2 u}{\partial t^2}(0,t) = -k\,u(0,t) + T_0\frac{\partial u}{\partial x}(0,t)}$$

m mass of the object attached at the endpoint of the string.

The simplest case is when assume $m=0$ "massless spring". In this case our B.C. is

$$-k\,u(0,t) + T_0 \frac{\partial u}{\partial x}(0,t) = 0$$

$k, T_0 > 0$ otherwise we have a non physical setting

$$\boxed{-u(0,t) + h\,\frac{\partial u}{\partial x}(0,t) = 0}$$

must have $h>0$

My interpretation of this B.C.

is as follows: ↓ this B.C is telling me

$$\text{BC}: \underbrace{\frac{\partial u}{\partial x}(0,t)}_{\text{slope}} = \underbrace{\frac{u(0,t)}{h}}_{\text{slope}}$$