The following

two ways of visualizing

$$x^2$$

will be useful

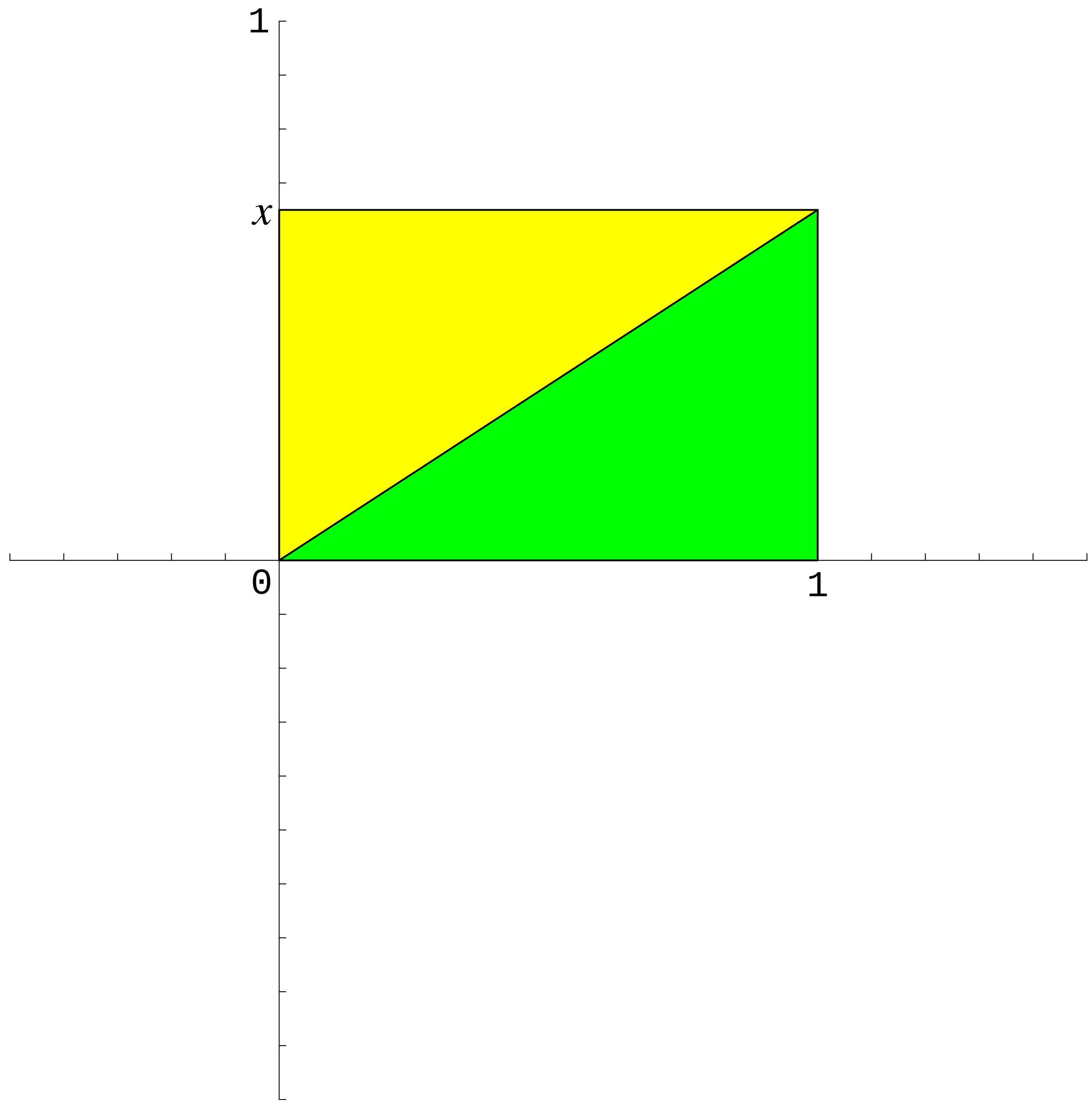
for Integral Calculus

Compare

the yellow area

and

the green area.

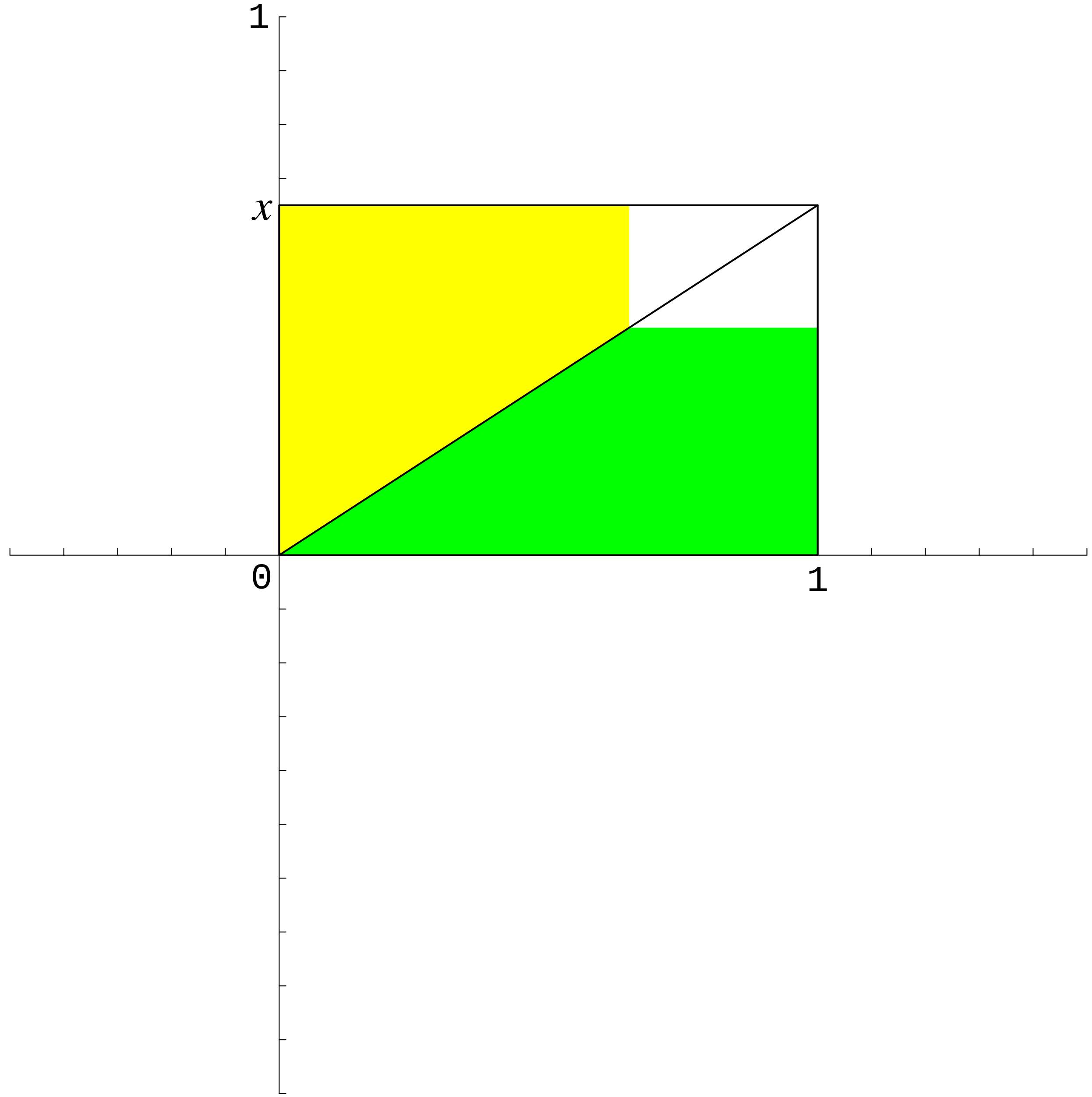


Compare

the yellow area

and

the green area.

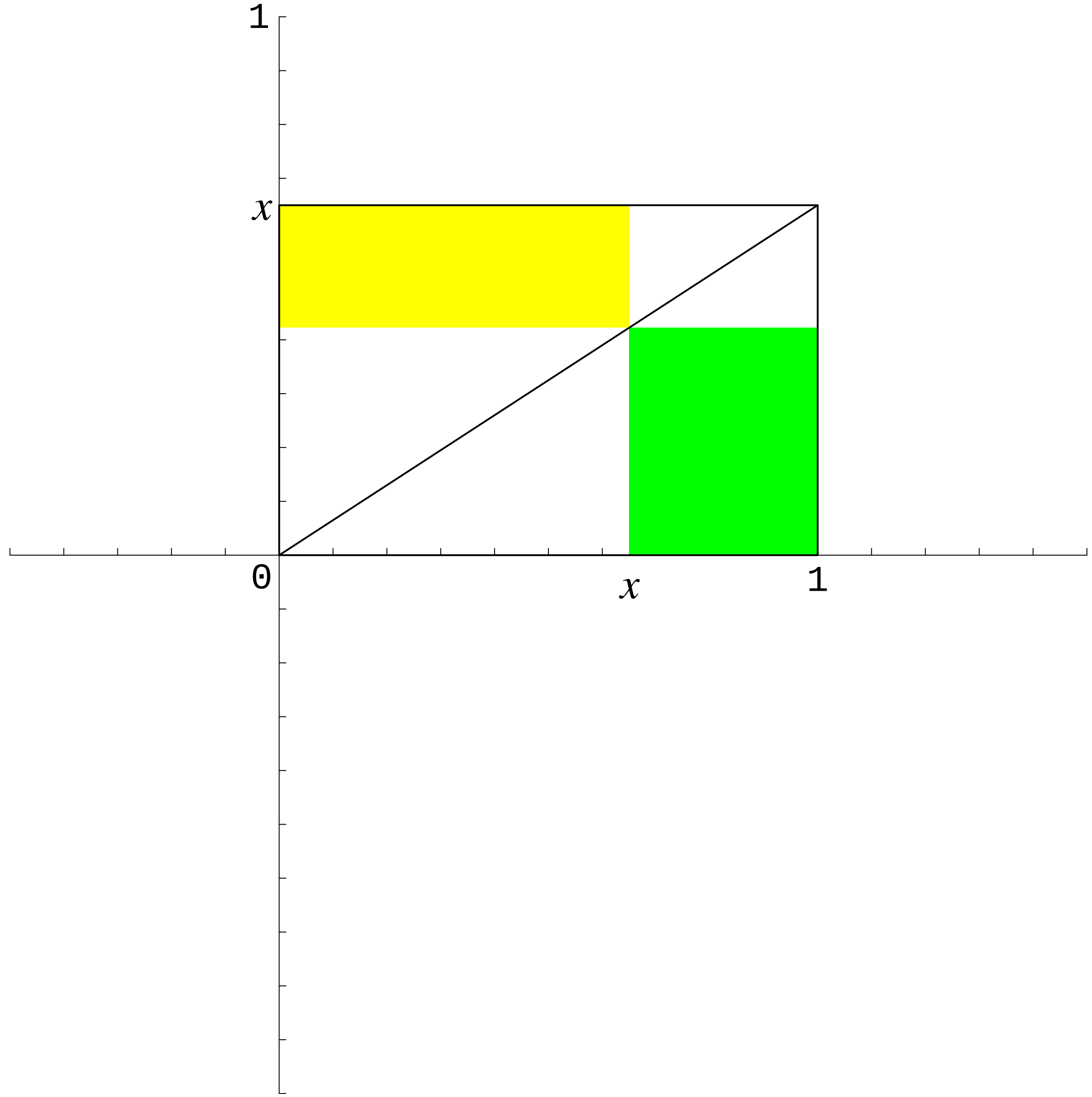


Compare

the yellow area

and

the green area.

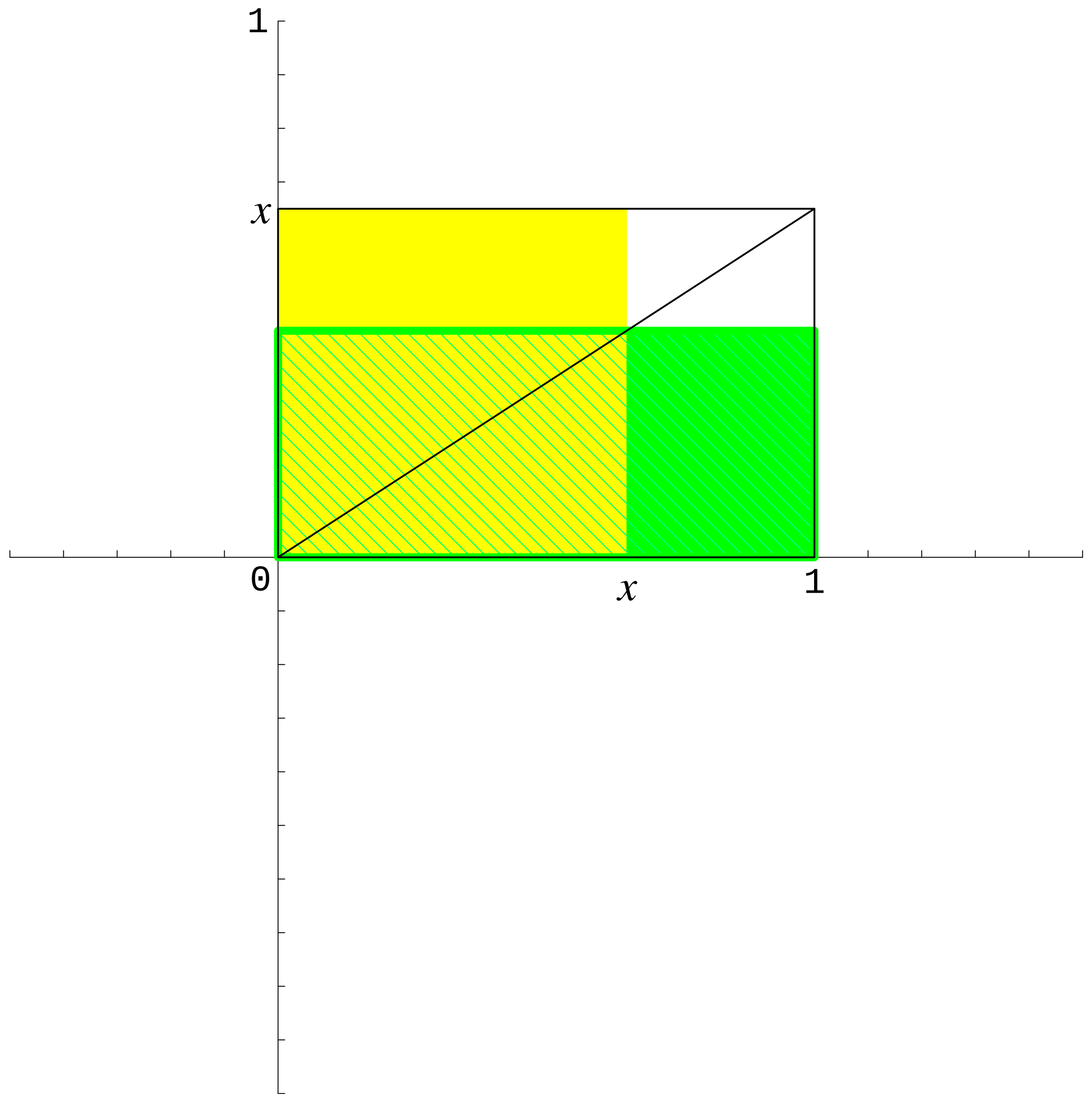


Compare

the yellow area

and

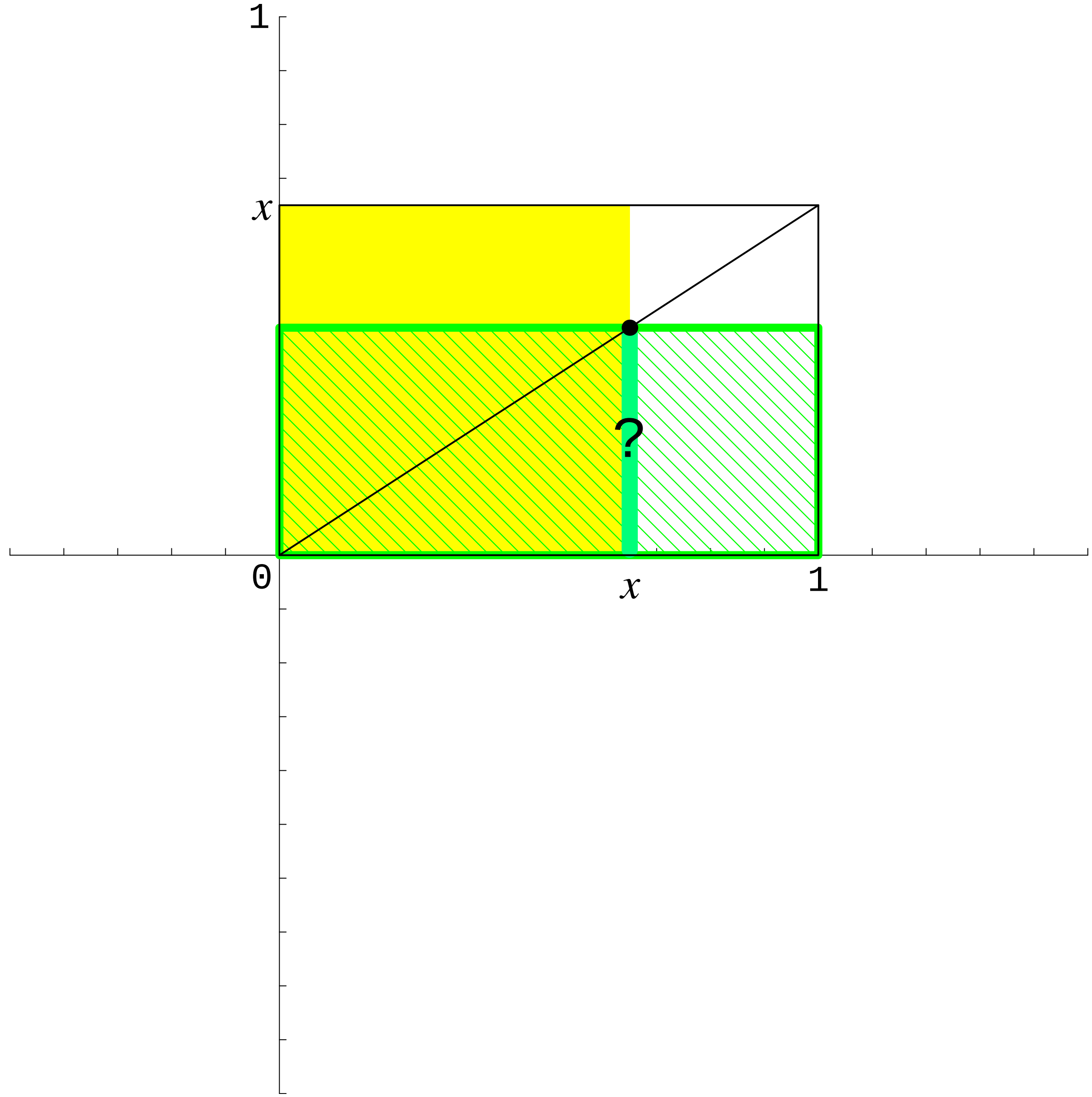
the green area.



How long is

the green

line-segment?

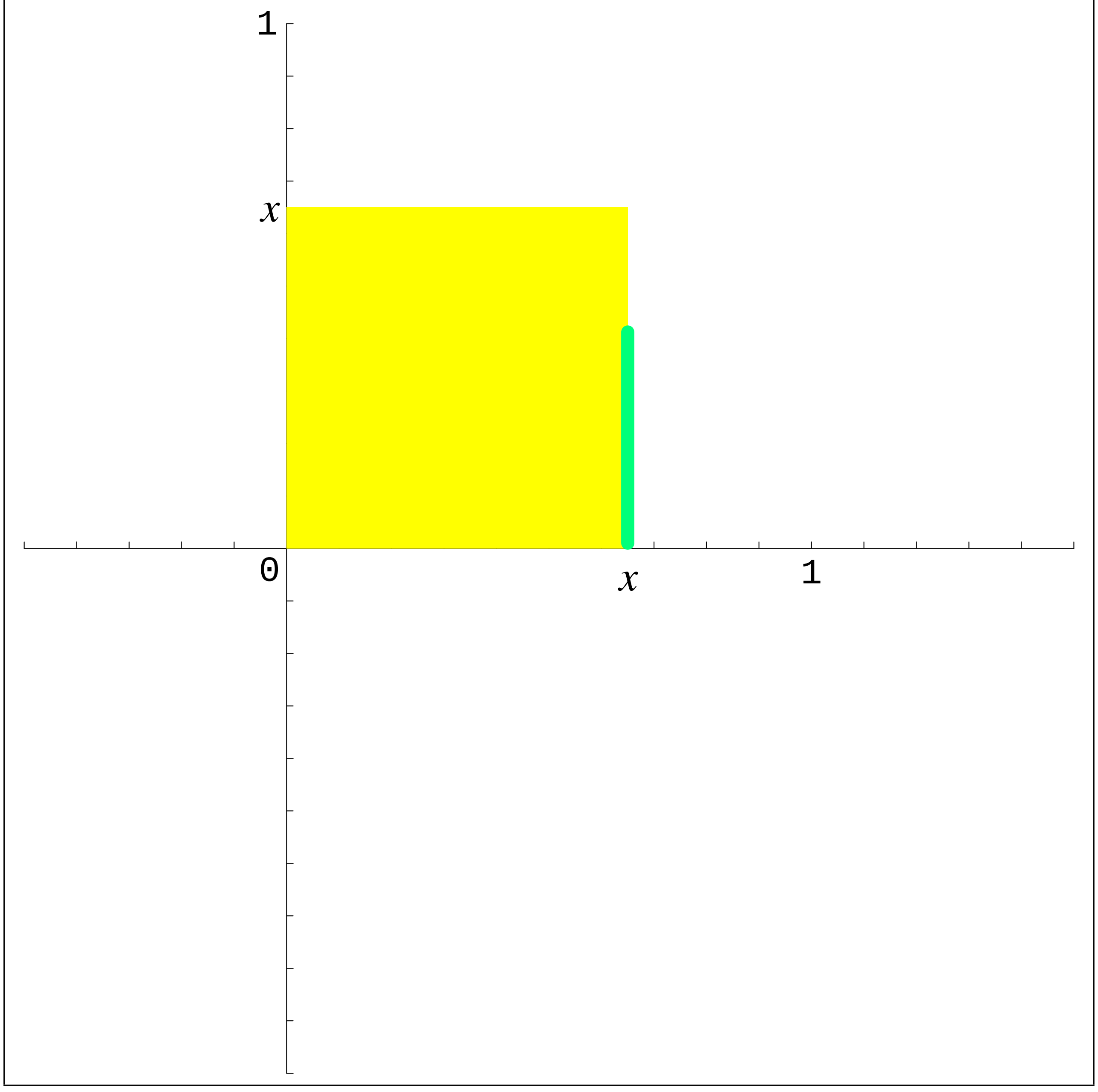


Two ways of visualizing x^2:

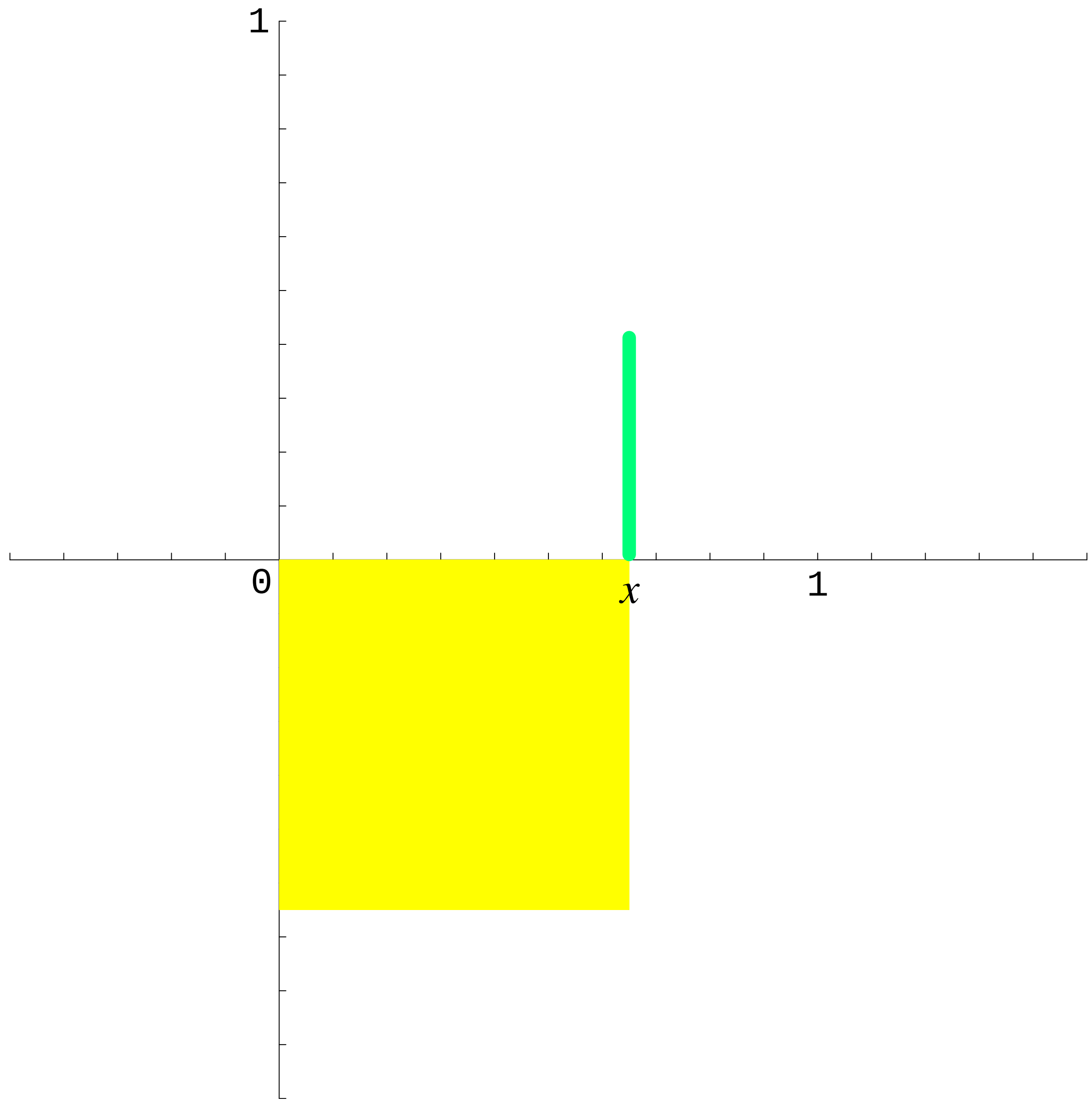
the yellow area

and

the green line-segment.

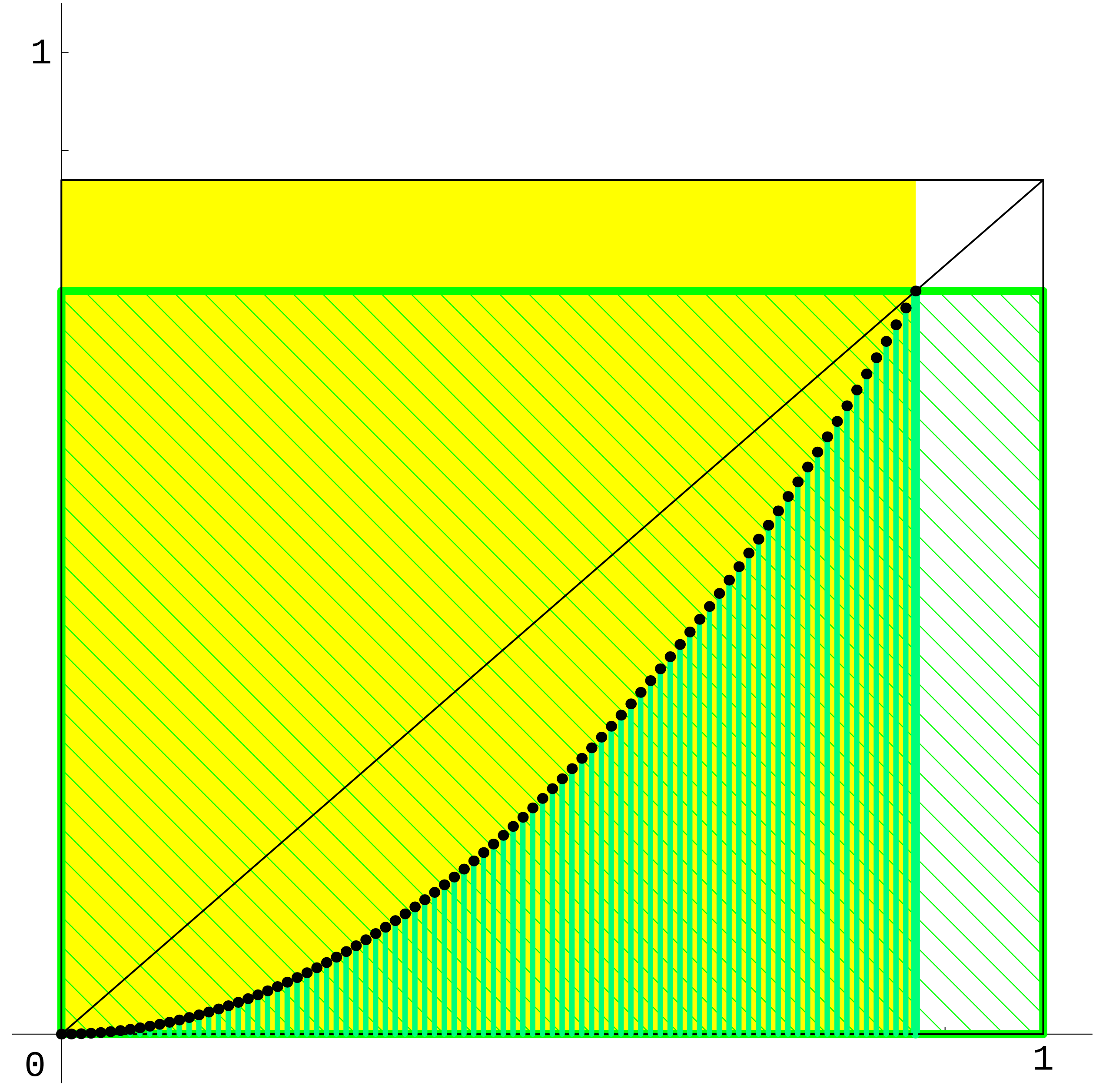


A similar picture will appear again, soon!



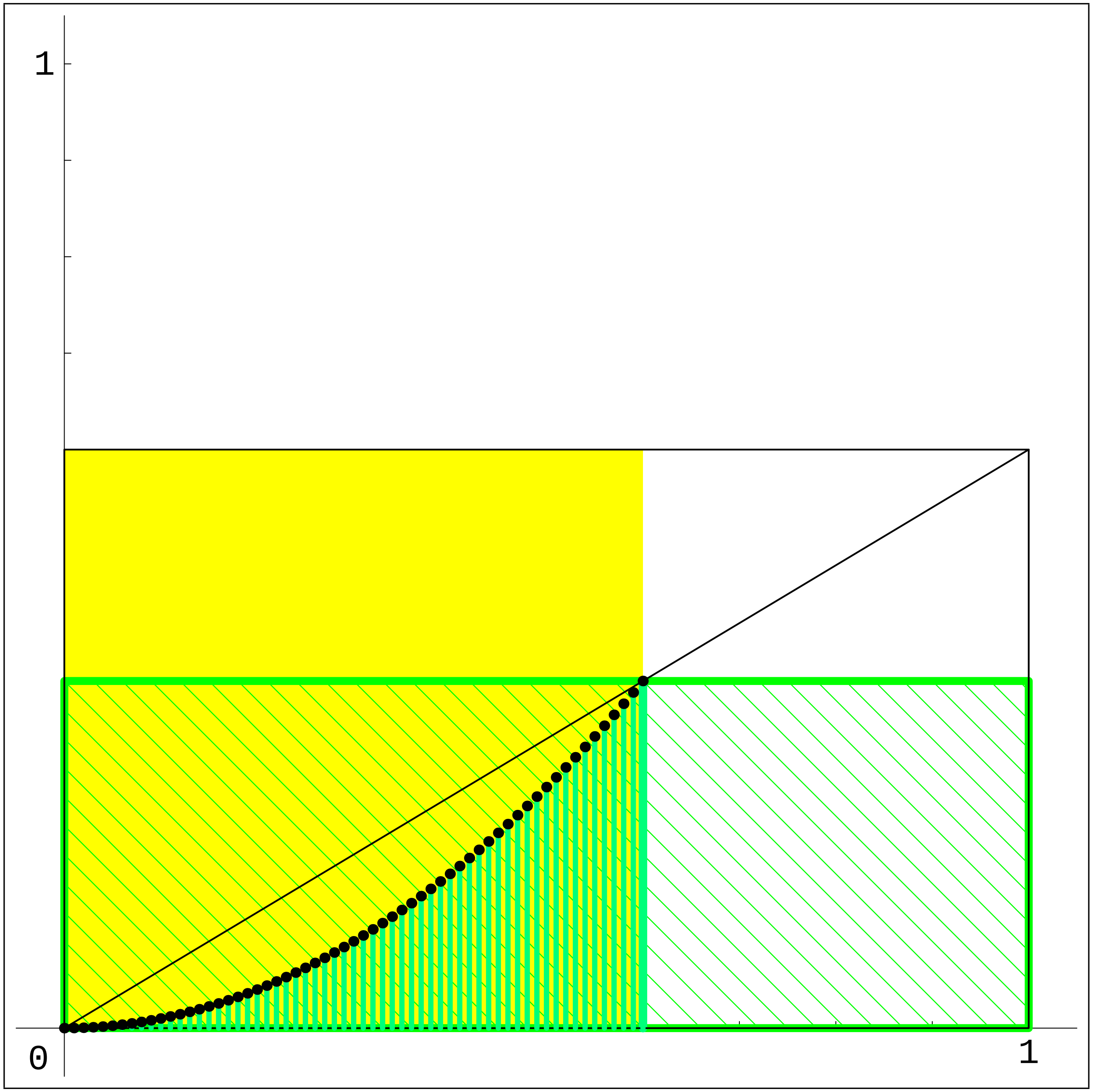
$$y = x^2$$

$$0 \leq x \leq 1$$



$$y = x^2$$

$$0 \le x \le 1$$



Now we are ready for

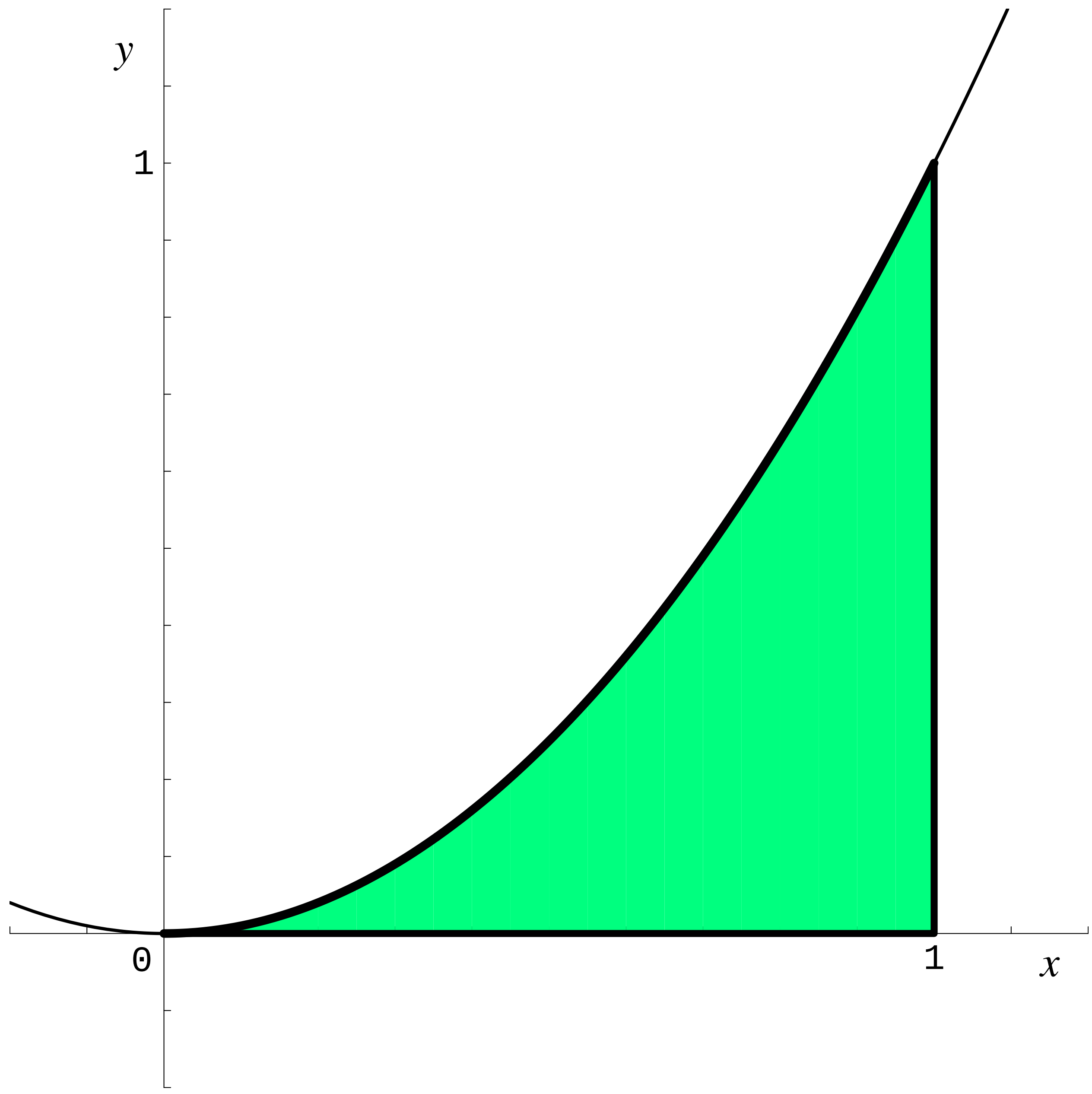
Integral Calculus

We consider

$$y = x^2$$

Our goal is to calculate the Spring-Green area.

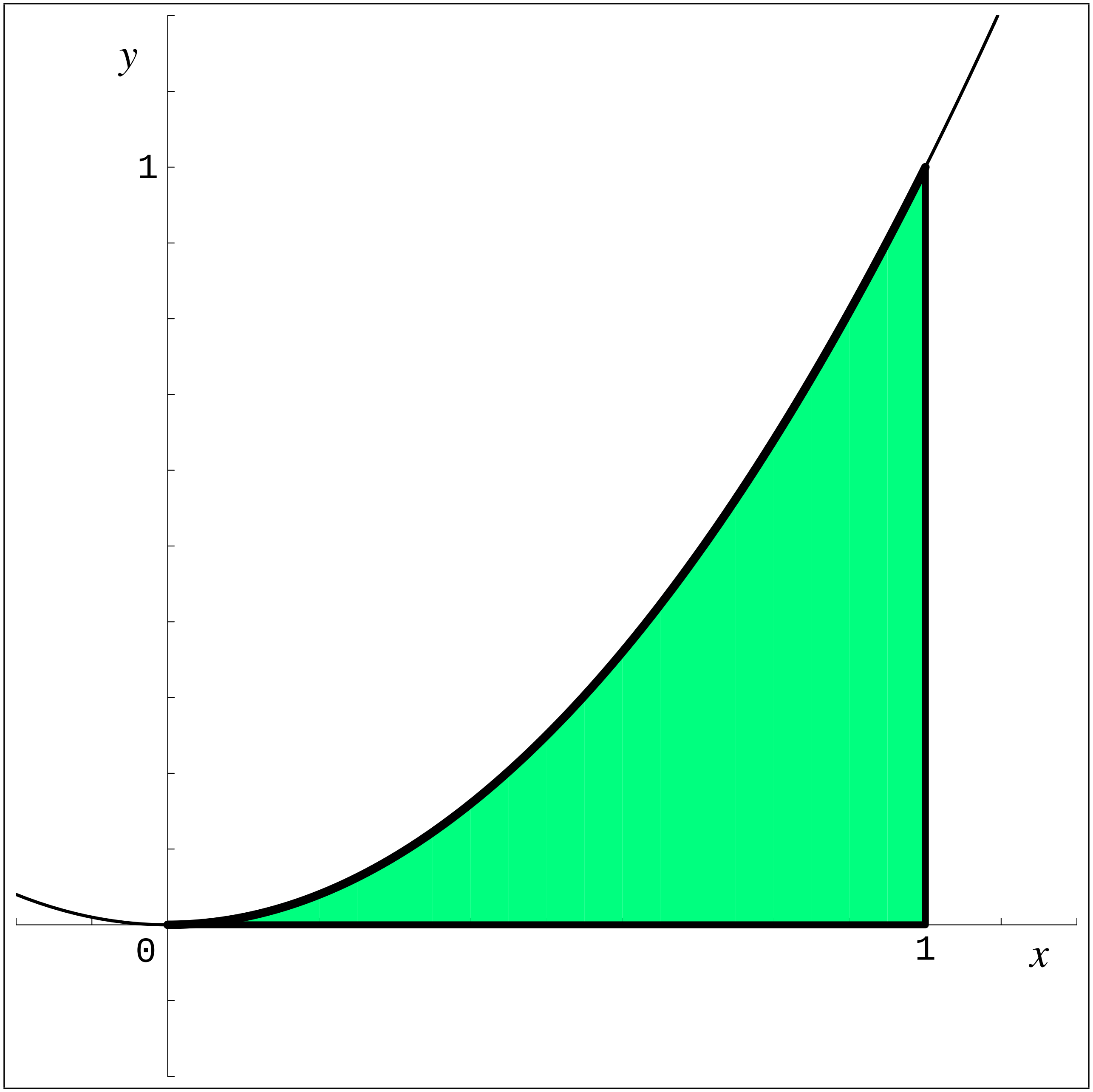
We will appeal to geometric intuition only.



We consider

$$y = x^2$$

This is a hard problem.



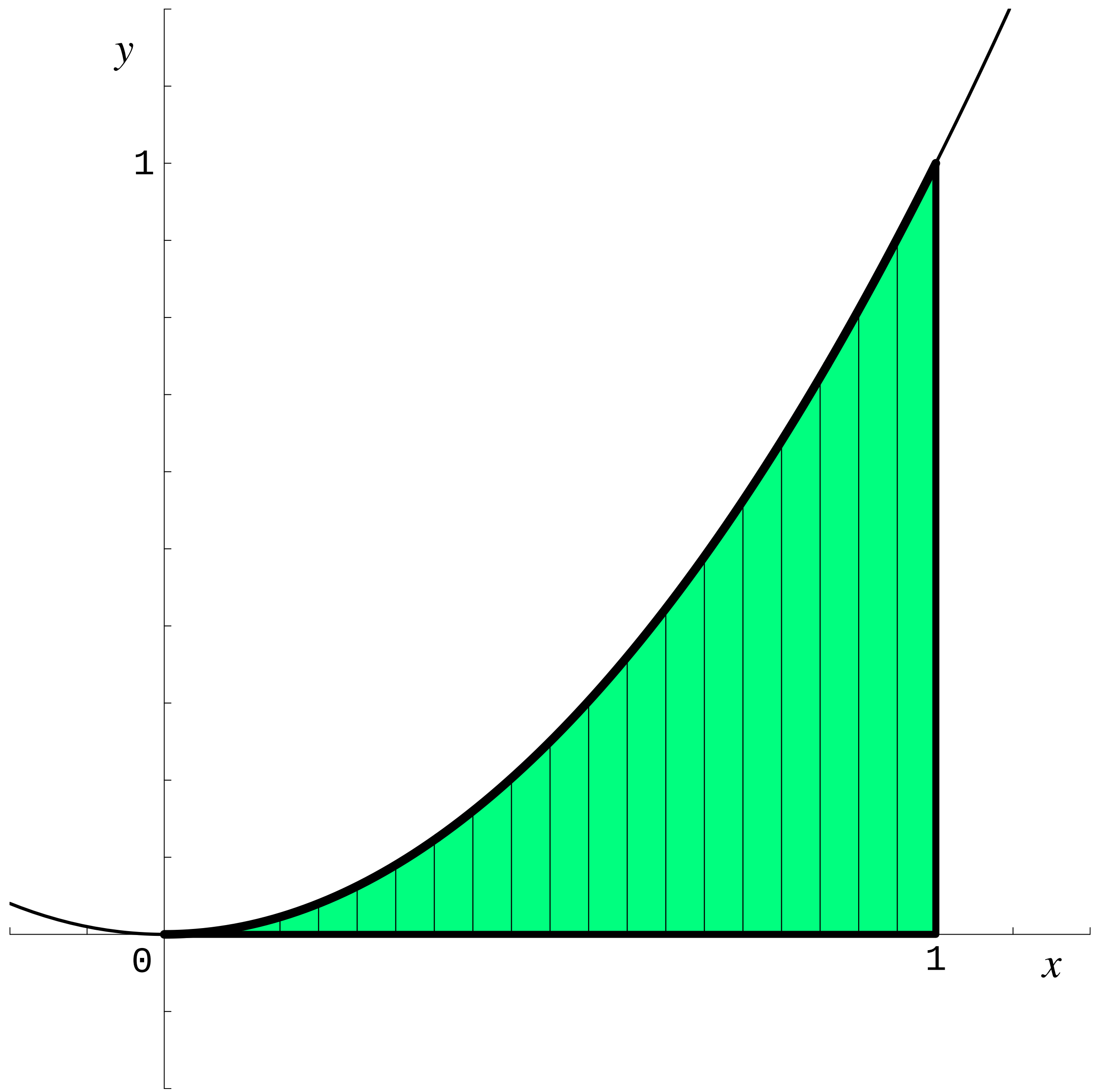
We consider

$$y = x^2$$

This is a hard problem.

Step 1.

We make the problem easier by considering many narrow strips.



We consider

$$y = x^2$$

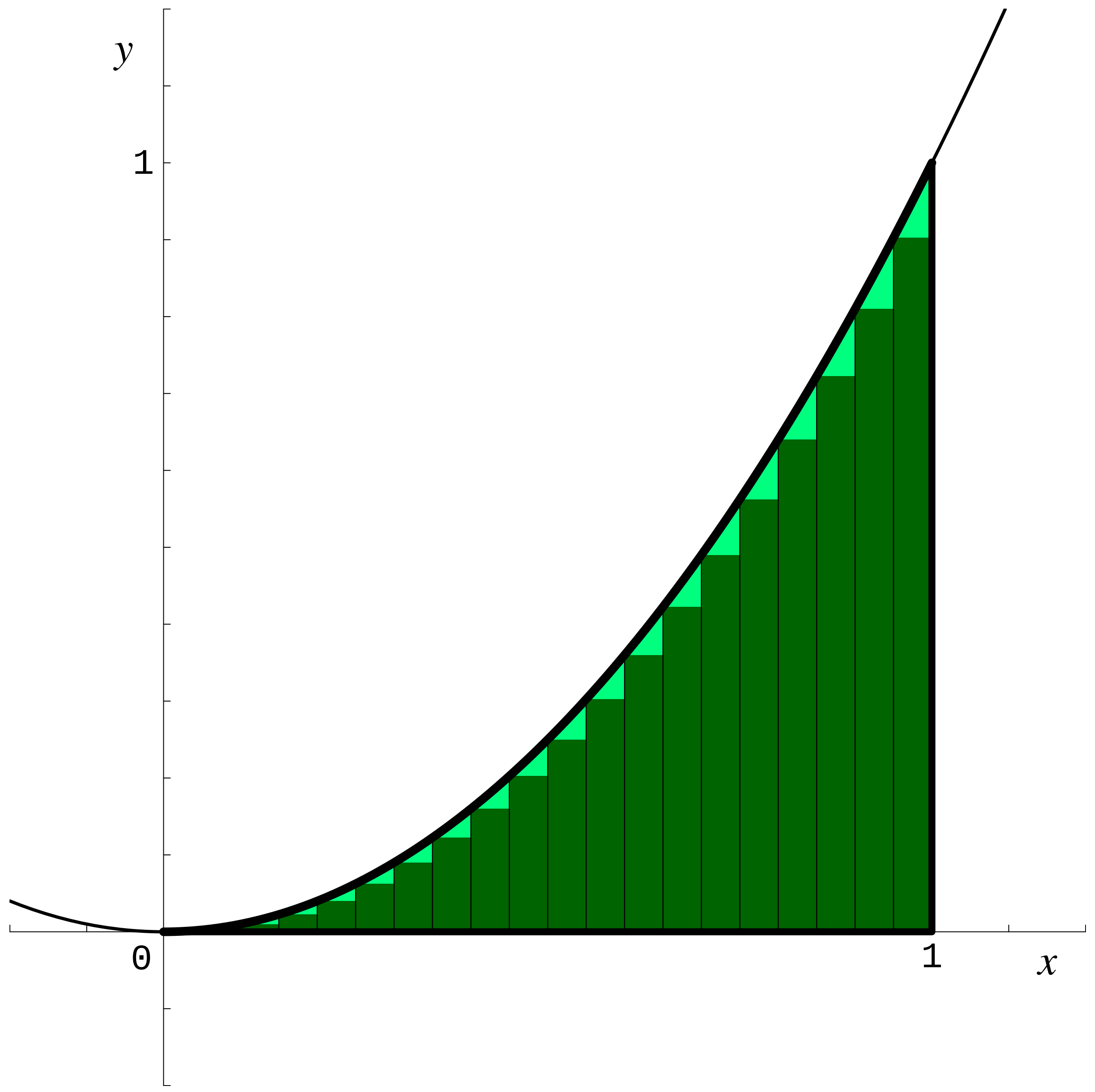
Step 2.

We approximate each narrow strip by a narrow rectangle; we call the narrow rectangles

linguini.

Note:

linguini:
plural of linguina



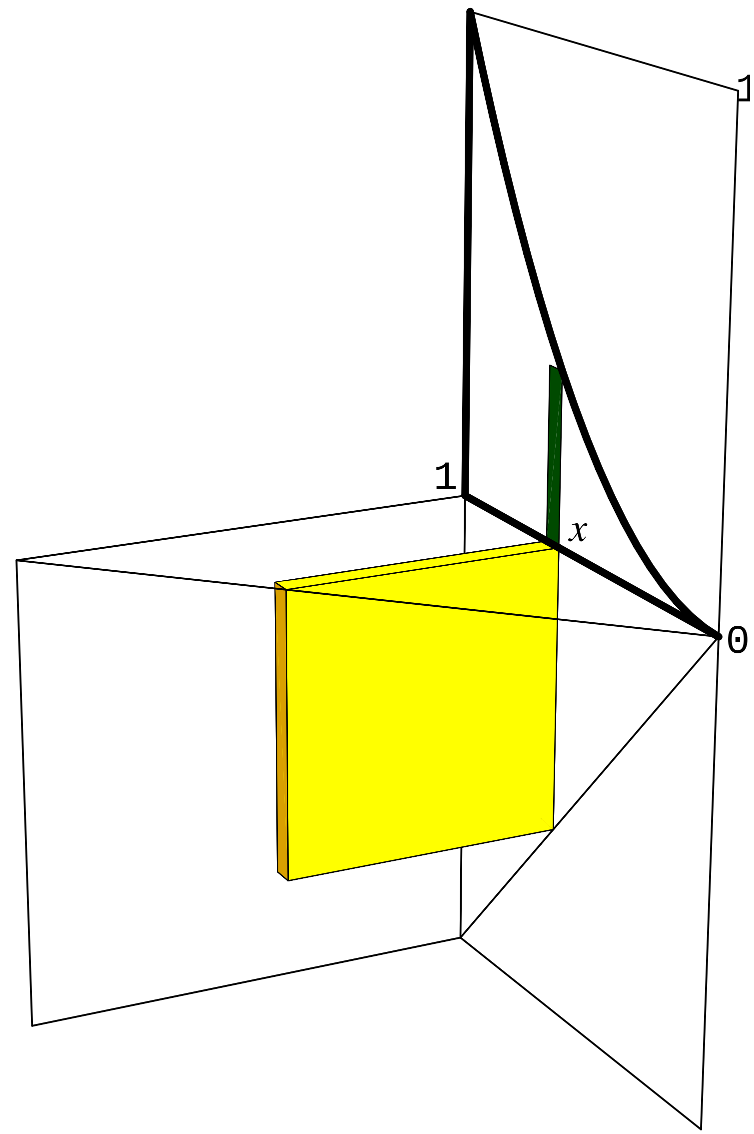
We consider

$$y = x^2$$

Step 3.

For each linguina
we construct
a thin box
with a square base
such that

$$\boxed{\text{Volume of Box}}$$
$$\|$$
$$\boxed{\text{Area of Linguina}}$$



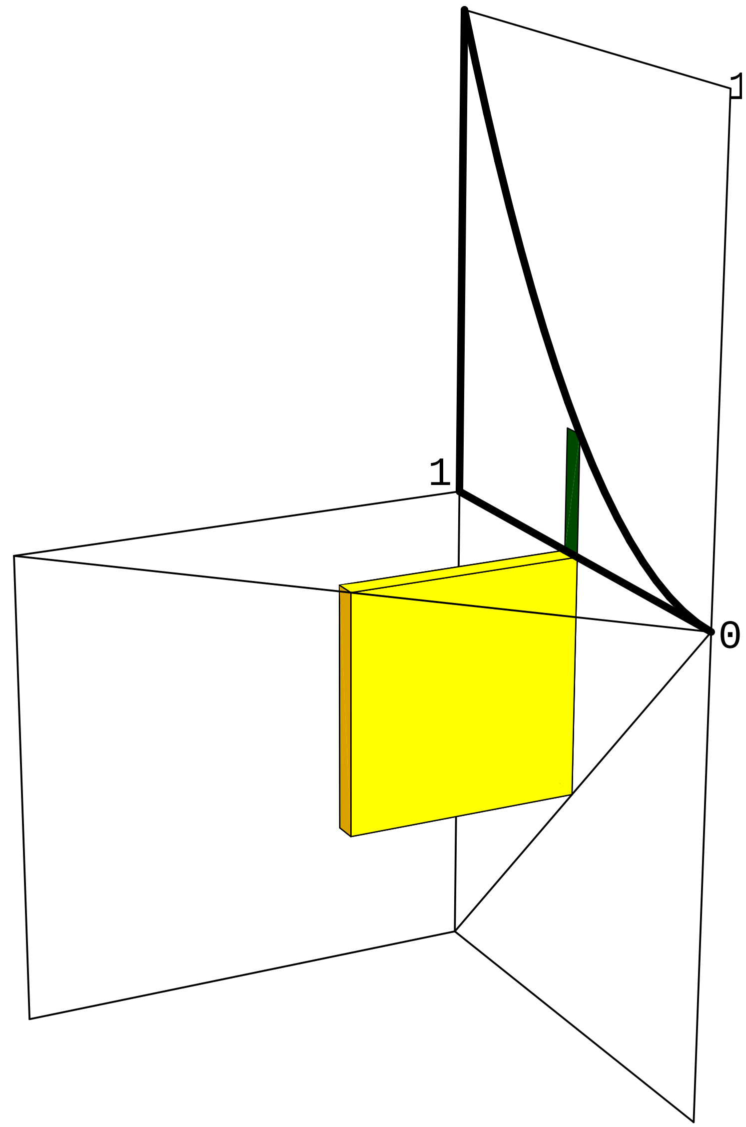
We consider

$$y = x^2$$

Step 4.

We construct
a thin box
for each linguina.

We construct
thin boxes
for the linguini.



We consider

$$y = x^2$$

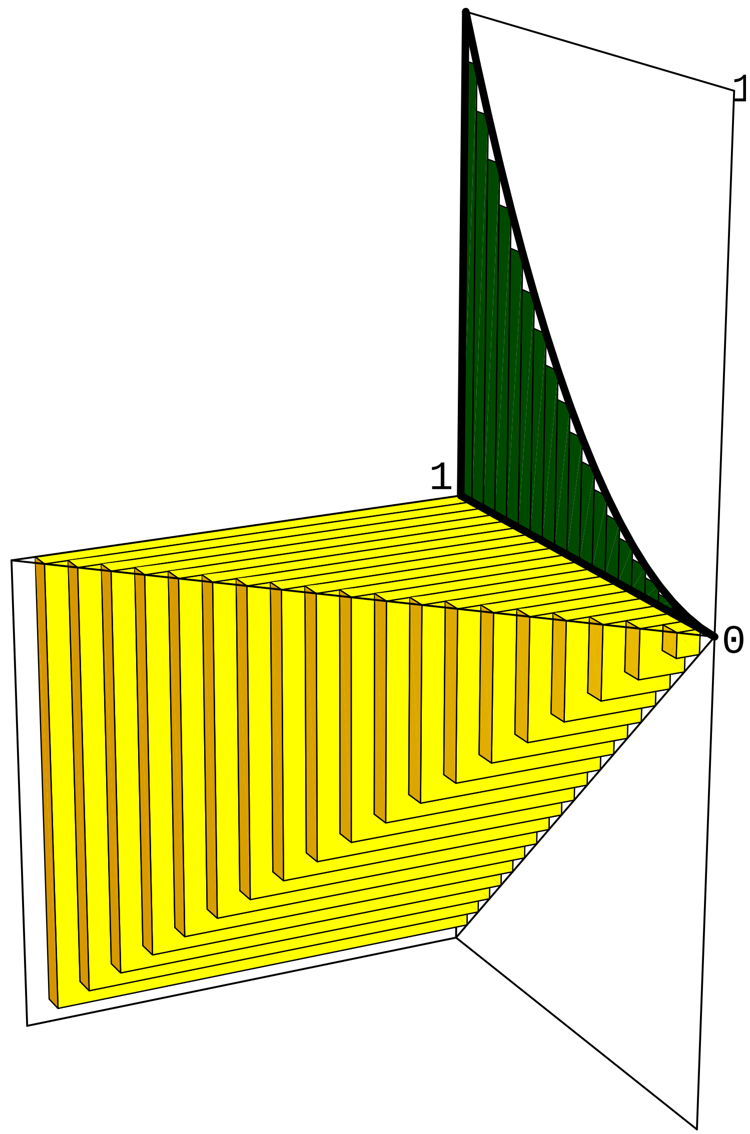
Step 5.

Put all
the thin boxes
together.

Vol. of all Boxes

||

Area of Linguini



We consider

$$y = x^2$$

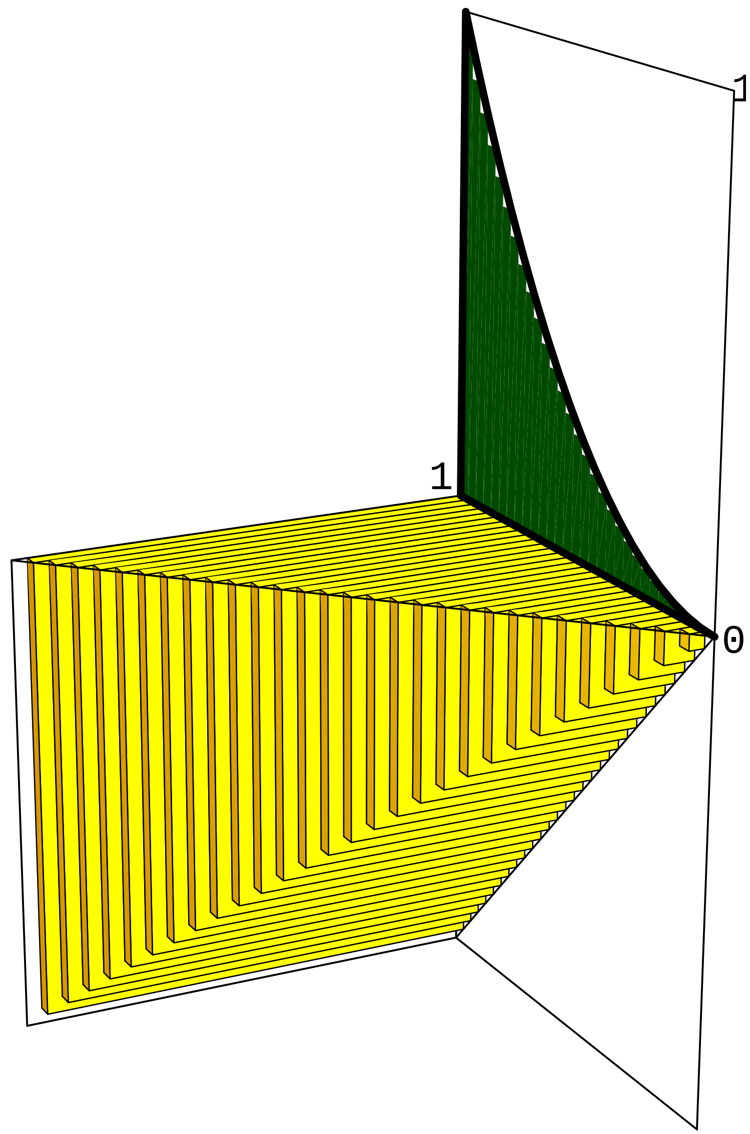
Step 6.

Go back to Step 1.
Make thinner
linguini.

Vol. of all Boxes

||

Area of Linguini



We consider

$$y = x^2$$

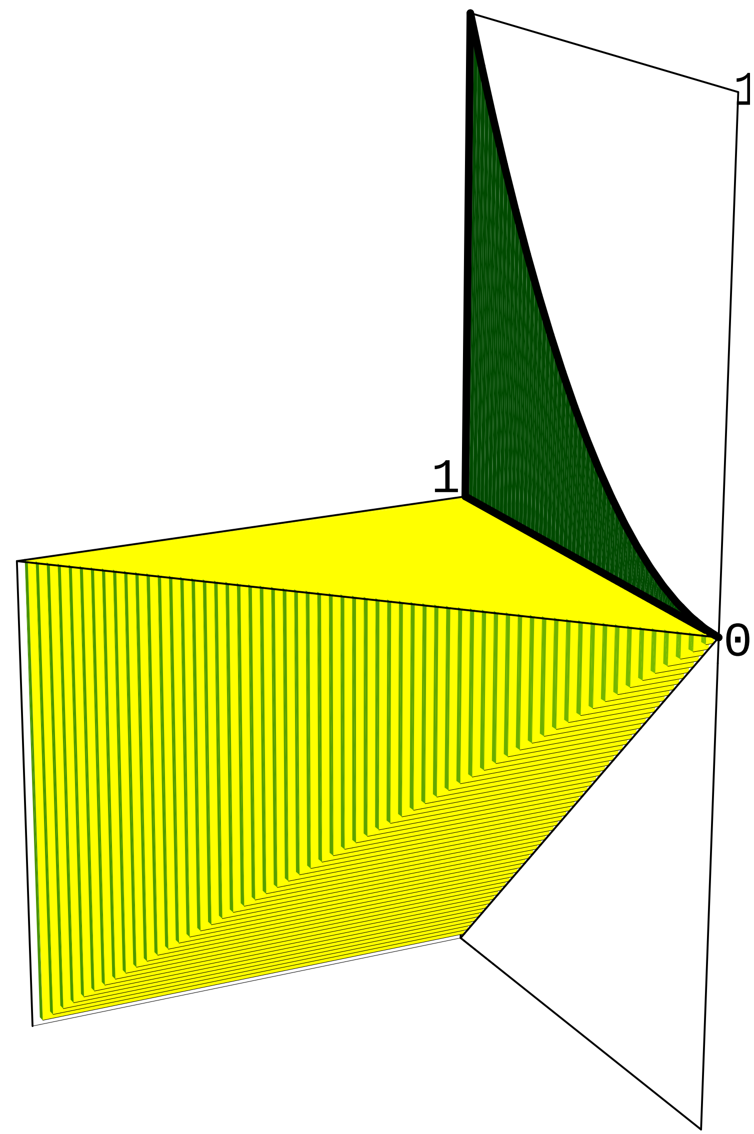
Step 7.

Go back to Step 1. Make even thinner linguini.

Vol. of all Boxes

$\|$

Area of Linguini



We consider

$$y = x^2$$

Step 8.

Boxes $\longrightarrow$ Pyramid

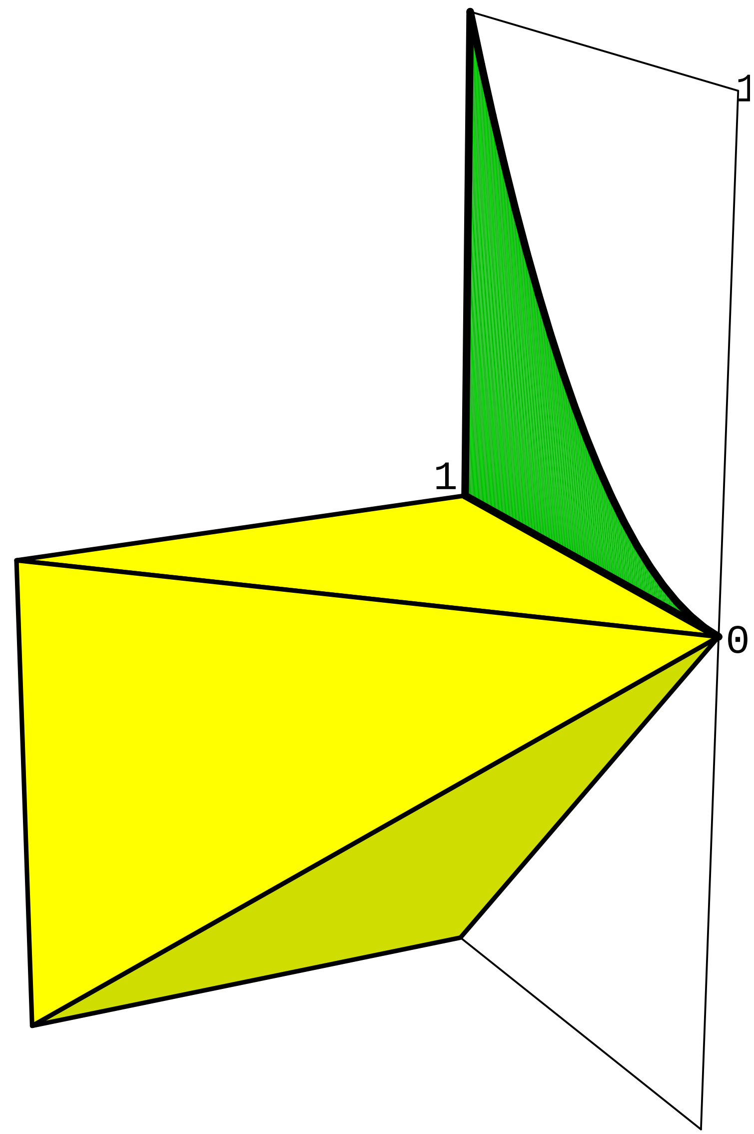
and

Linguini $\longrightarrow$ Area

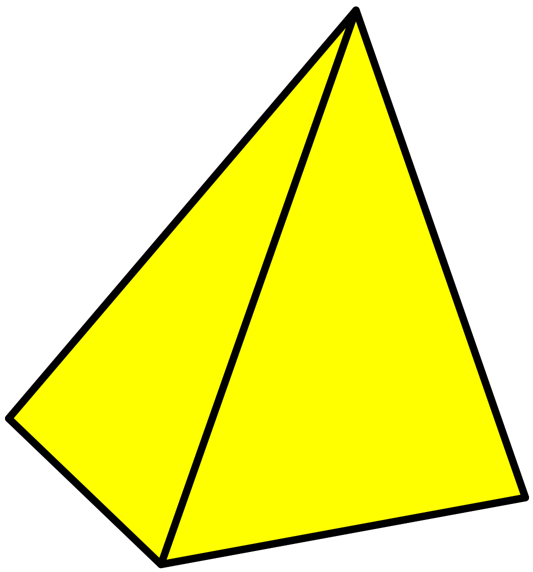
Vol. of Pyramid

$\|$

Spring-Green Area

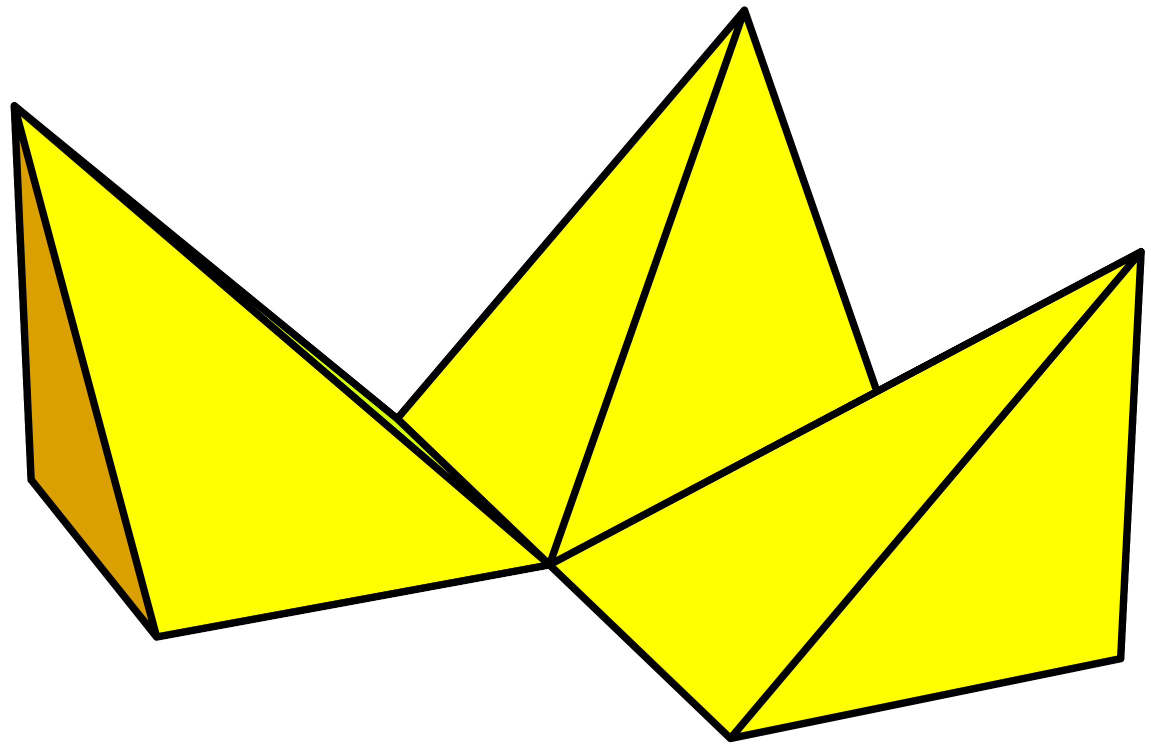


What is the volume of the yellow pyramid?

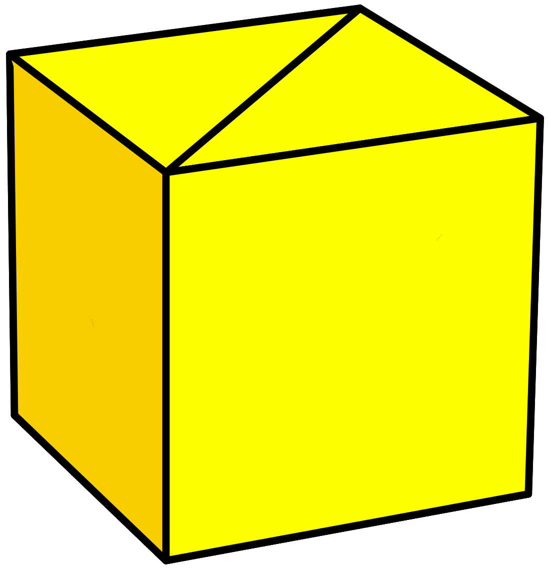


What is the volume of the yellow pyramid?

It is easier to consider three yellow pyramids.



... and set the three pyramids in motion ...

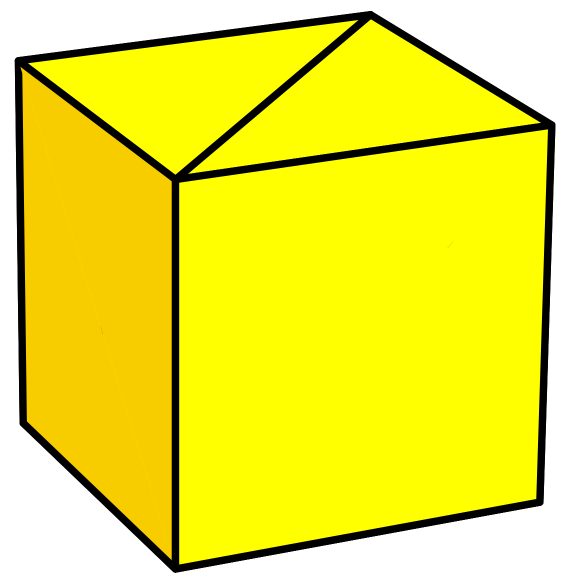


Now it is clear that the volume of the three pyramids is

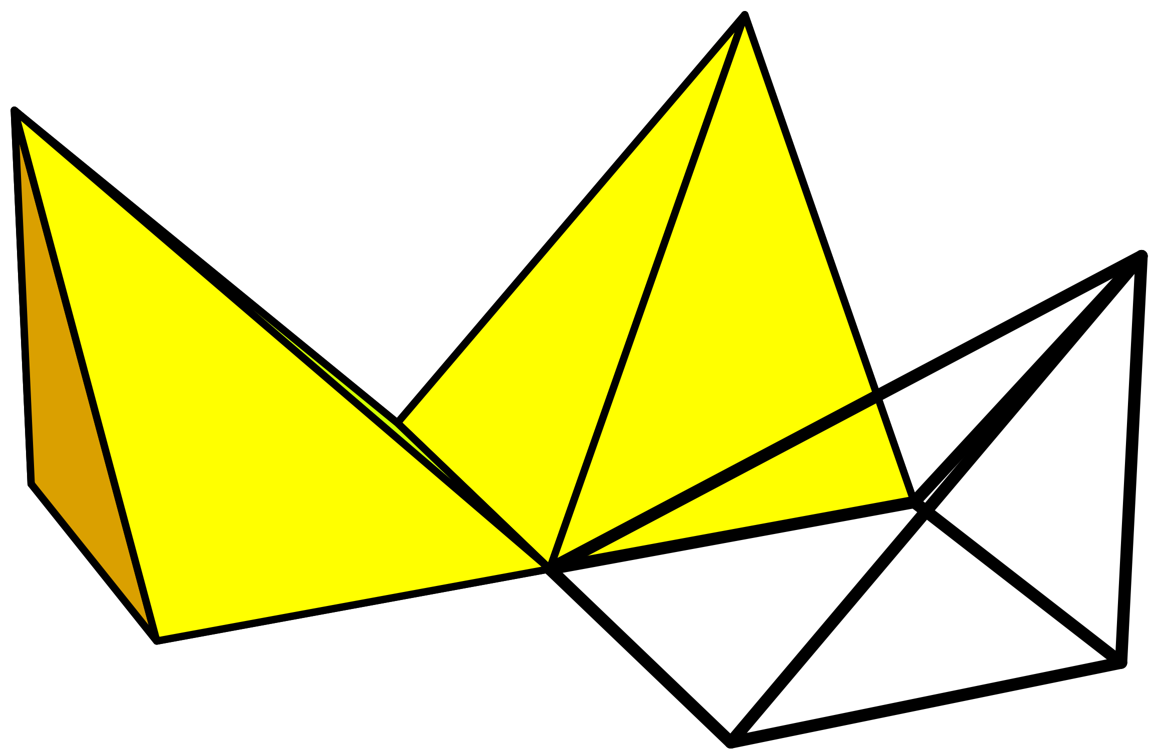
$$1.$$

Hence, the volume of the yellow pyramid is

$$\frac{1}{3}.$$

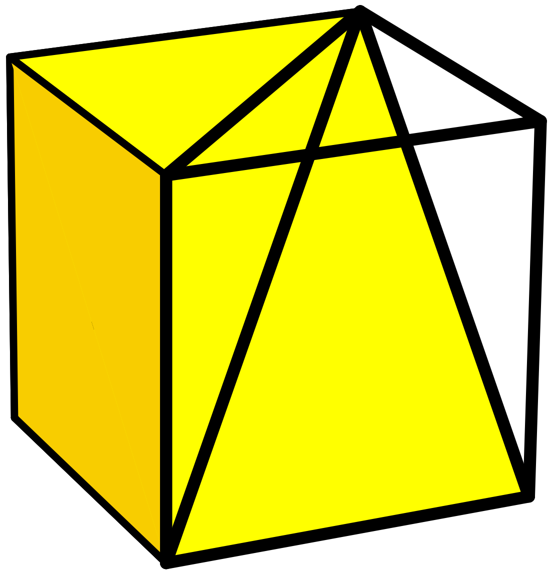


Or, let one pyramid be transparent.



... and set them

in motion ...

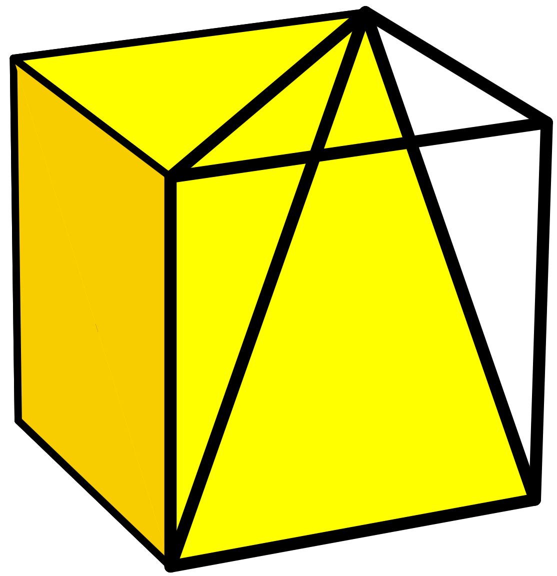


You can make

this model

using the

handouts.



We consider

$$y = x^2$$

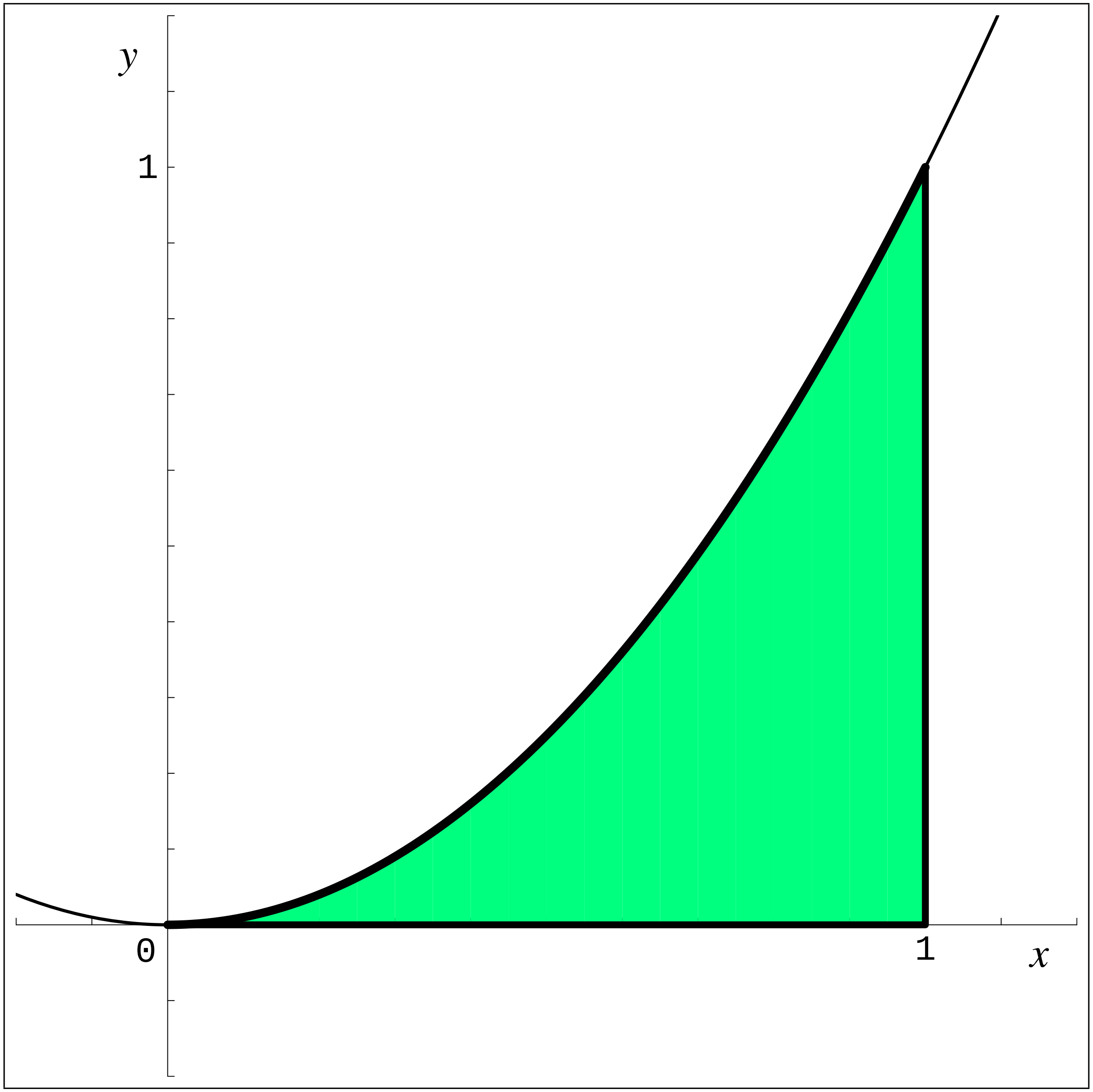
Finally,

we conclude that

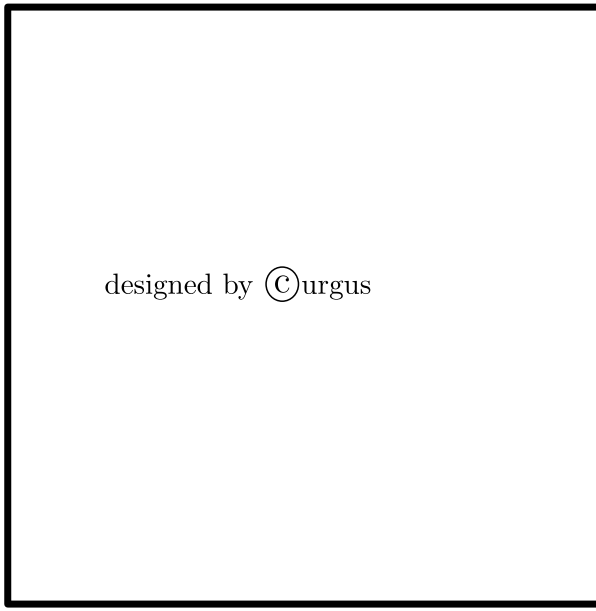
the Spring-Green

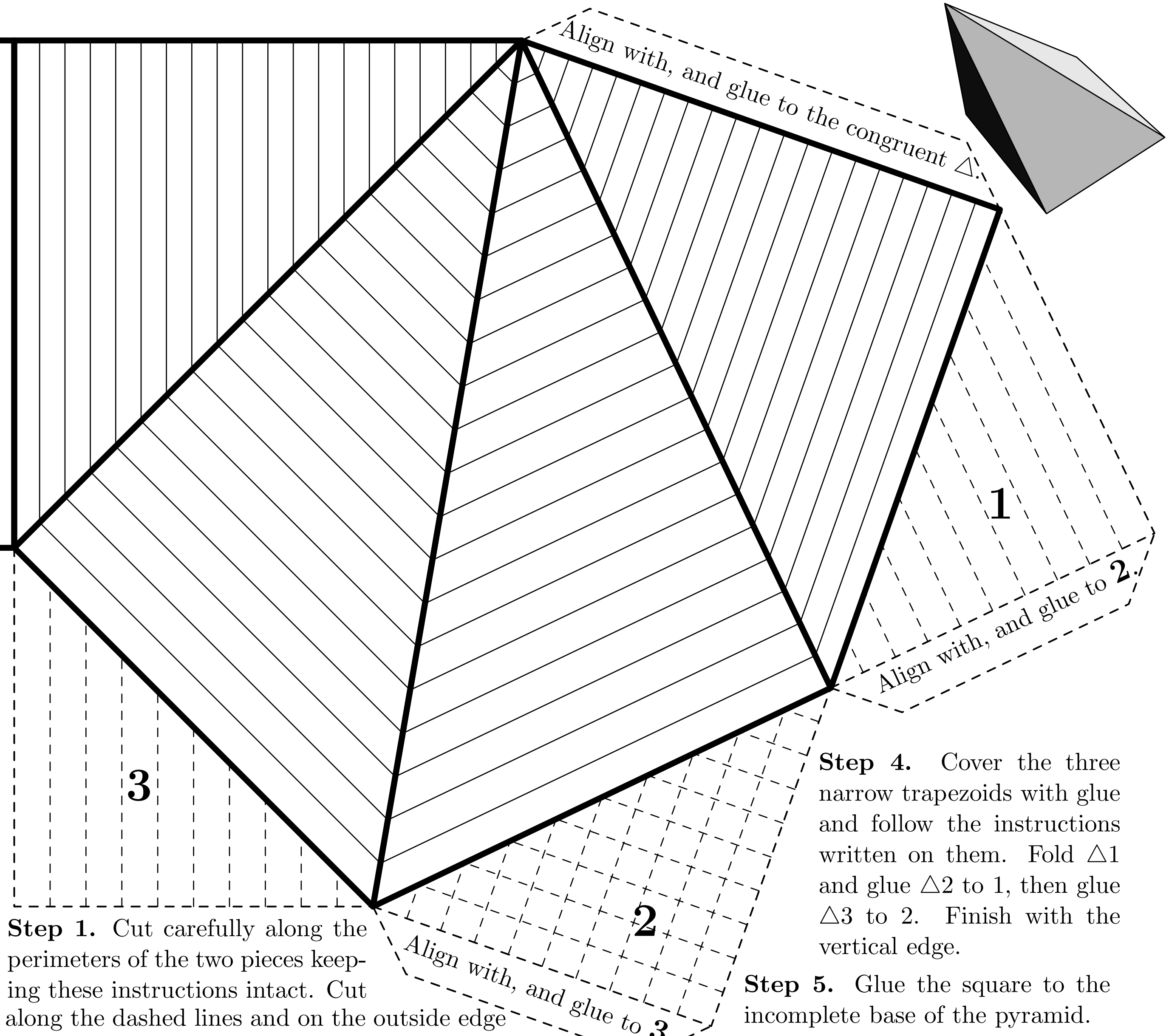
area equals

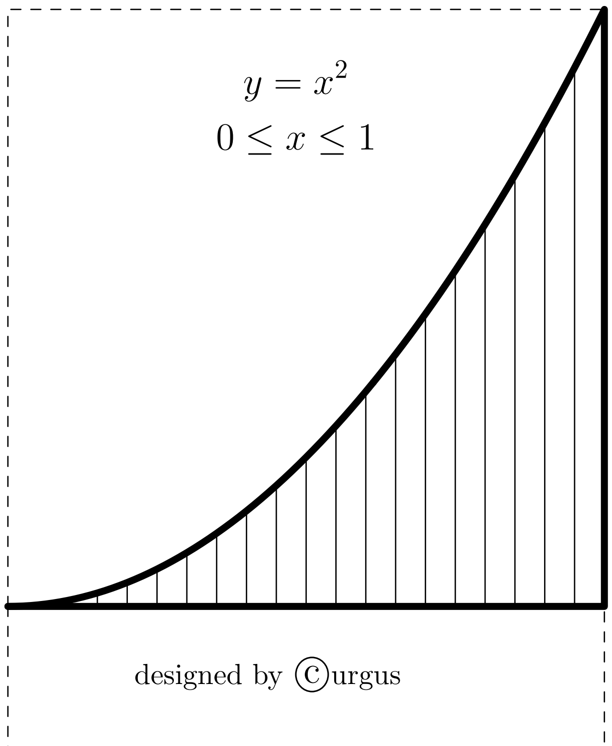
$$\frac{1}{3}$$



the end







Step 1. Cut carefully along the perimeters of the two pieces keeping these instructions intact. Cut along the dashed lines and on the outside edge of the thick lines. (The piece with the parabola will be used in Step 6.)

Step 2. Identify the **folding lines**: the thick black interior lines on the bigger piece. Enjoy their lengths: five lengths 1, two lengths $\sqrt{2}$, and one $\sqrt{3}$.

Step 3. Press all folding lines with a black ballpoint pen to ensure easy folding. (Use a ruler.) Fold along all folding lines keeping the printed side on the outside. No folding along dashed lines.

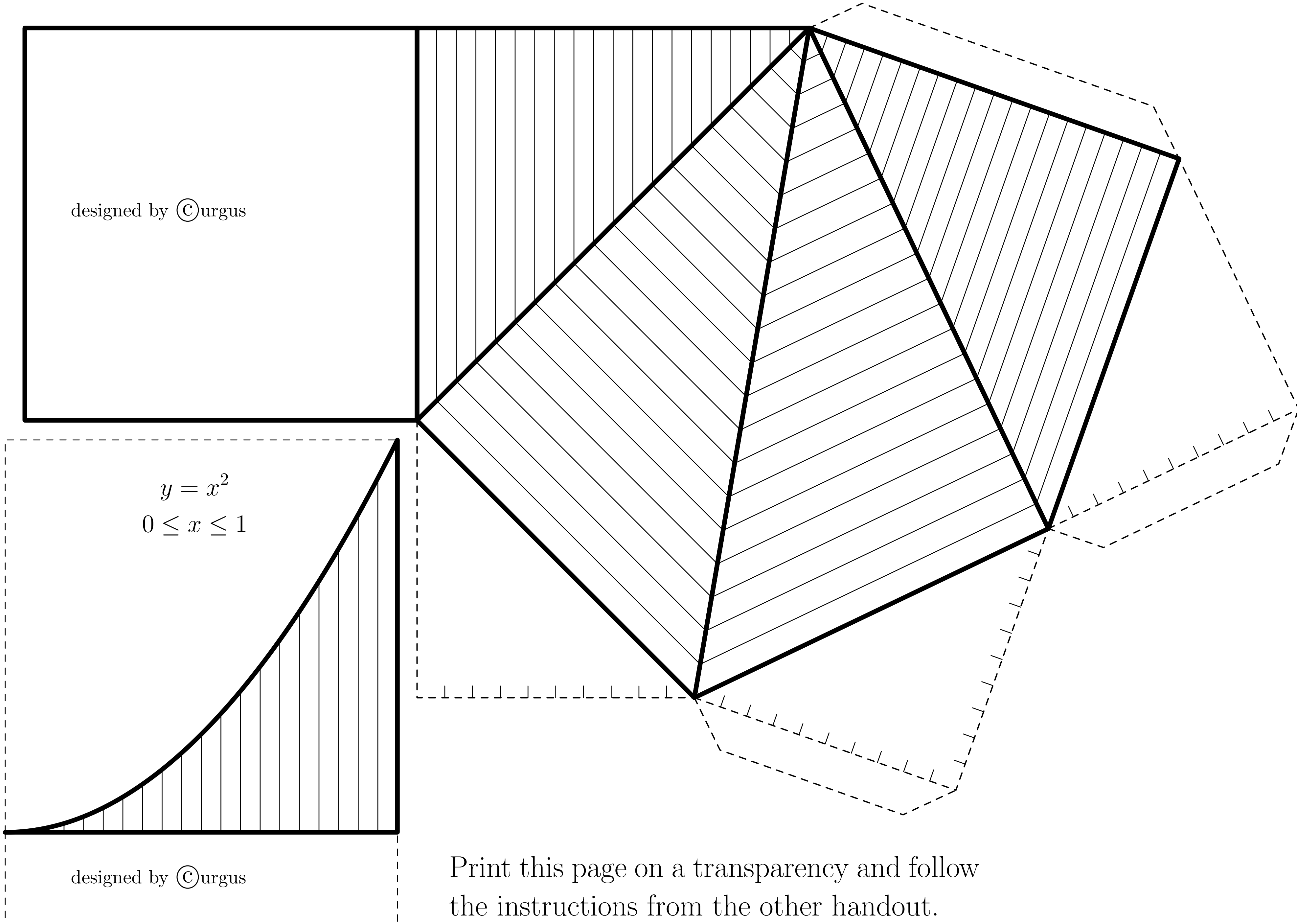
Step 4. Cover the three narrow trapezoids with glue and follow the instructions written on them. Fold △1 and glue △2 to 1, then glue △3 to 2. Finish with the vertical edge.

Step 5. Glue the square to the incomplete base of the pyramid.

Step 6. Notice that the folding lines from Step 2 are now the edges of the pyramid. Again, enjoy their lengths: five lengths 1, two lengths $\sqrt{2}$, and one $\sqrt{3}$.

Step 7. Verify that three pyramids form a cube.

Step 8. Understand why the volume of the pyramid and the area of the striped region underneath the parabola are equal.



Print this page on a transparency and follow the instructions from the other handout.