I like to know where my notebook resides.

In[1]:=
```
NotebookDirectory[]
```

Out[1]=
```
C:\Dropbox\Work\myweb\Courses\Math_pages\Math_430\
```

# Natural Modes of Vibration

Below are the natural modes of vibration of a string with the mass density $\rho$, with the magnitude of the tensile force $T_0$ and the length lL. We consider two different time parts, cosine and sine:

In[2]:=
```
Clear[NMVC, NMVS, x, t, n, ρ, T0, lL];
```

$$\texttt{NMVC[x\_, t\_, n\_, T0\_, }\rho\texttt{\_, lL\_] := Sin}\left[\frac{\texttt{n Pi}}{\texttt{lL}}\,\texttt{x}\right]\texttt{ Cos}\left[\frac{\texttt{n Pi}\sqrt{\texttt{T0}}}{\sqrt{\rho}\,\texttt{lL}}\,\texttt{t}\right];$$

$$\texttt{NMVS[x\_, t\_, n\_, T0\_, }\rho\texttt{\_, lL\_] := Sin}\left[\frac{\texttt{n Pi}}{\texttt{lL}}\,\texttt{x}\right]\texttt{ Sin}\left[\frac{\texttt{n Pi}\sqrt{\texttt{T0}}}{\sqrt{\rho}\,\texttt{lL}}\,\texttt{t}\right]$$

Test the functions defined above:

In[5]:=
```
NMVC[x, t, 1, 1, 1, Pi]
```

Out[5]=
```
Cos[t] Sin[x]
```

In[6]:=
```
NMVS[x, t, 1, 1, 1, Pi]
```

Out[6]=
```
Sin[t] Sin[x]
```

To display the time nicely, I need to understand the command which controls the display of decimal numbers.

In[7]:=
```
? NumberForm
```

Out[7]=

Symbol

NumberForm[*expr*, *n*] prints with approximate real numbers in *expr* given to *n*-digit precision.

NumberForm[*expr*, {*n*, *f*}] prints with approximate real numbers having *n* digits, with *f* digits to the right of the decimal point.

NumberForm[*expr*] prints using the default options of NumberForm.

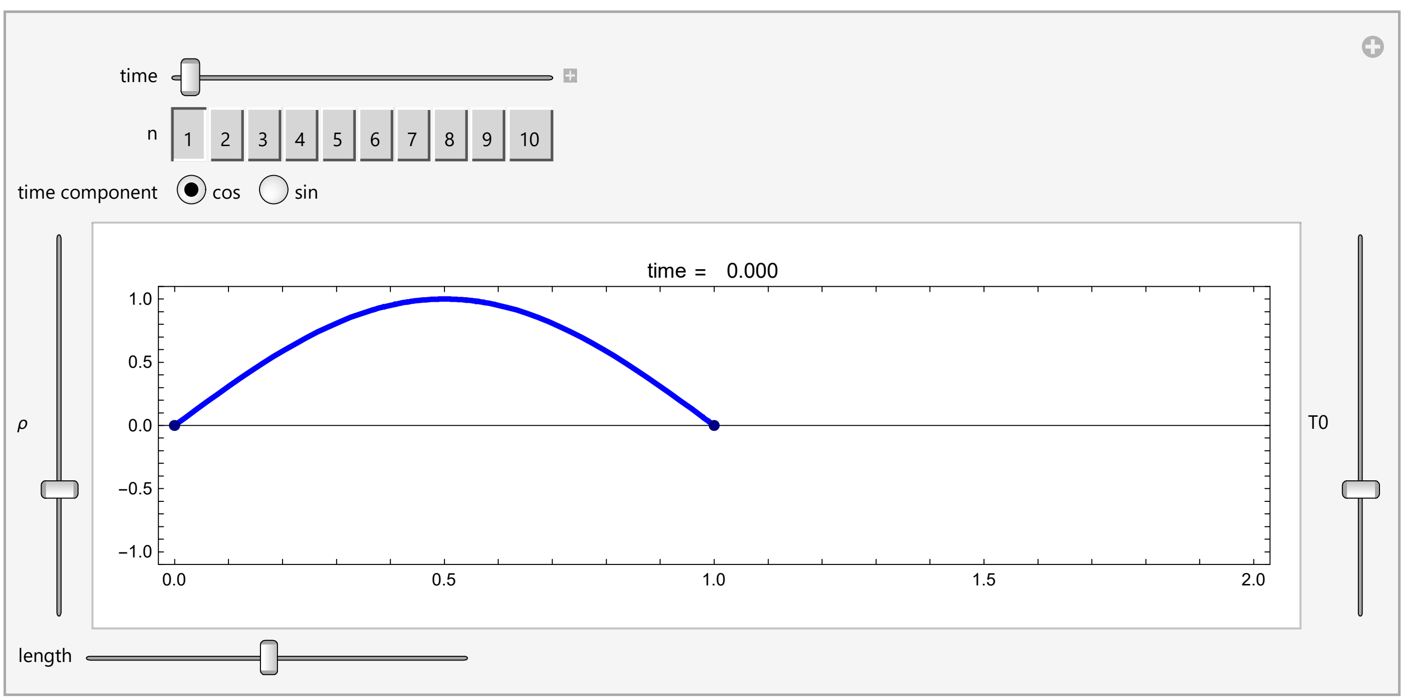
In[8]:=
```
NumberForm[N[Pi], {10, 9}]
```

Out[8]//NumberForm=
```
3.141592654
```

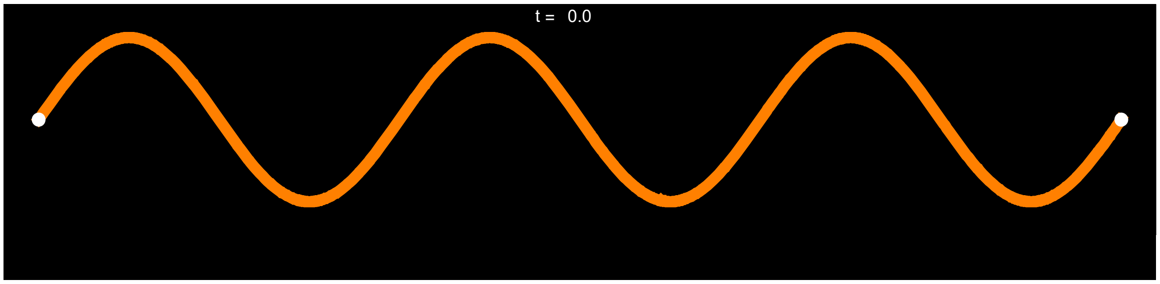
In[9]:=
```
Clear[T0, lL, tc]; Manipulate[
 Which[tc == "cos",
  Plot[
   Evaluate[NMVC[x, t, n, T0, ρ, lL]], {x, 0, lL},
   PlotStyle → {{Blue, Thickness[0.005]}},
   Epilog → {RGBColor[0, 0, .5], PointSize[0.01], Point[{0, 0}], Point[{lL, 0}]},
   PlotRange → {{-0.03, 2.03}, {-1.1, 1.1}},
   PlotLabel → TableForm[
     {"time", "=",
      NumberForm[N[t], {5, 3}, NumberPadding → {" ", "0"}]},
     TableDirections → Row, TableAlignments → {Left, Left, Right}, TableSpacing → {0.5, .3}],

   Frame → True, AspectRatio → 1 / 4, ImageSize → 600
  ],
  tc == "sin", Plot[
   Evaluate[NMVS[x, t, n, T0, ρ, lL]], {x, 0, lL},
   PlotStyle → {{Blue, Thickness[0.005]}},
   Epilog → {RGBColor[0, 0, .5], PointSize[0.01], Point[{0, 0}], Point[{lL, 0}]},
   PlotRange → {{-0.03, 2.03}, {-1.1, 1.1}},
   PlotLabel → TableForm[
     {"time", "=",
      NumberForm[N[t], {5, 3}, NumberPadding → {" ", "0"}]},
     TableDirections → Row, TableAlignments → {Left, Left, Right}, TableSpacing → {0.5, .3}],
   Frame → True, AspectRatio → 1 / 4, ImageSize → 600
  ]
 ]
 ,
 {{t, 0, "time"}, 0, 8 Pi, N[Pi/128]}, {{n, 1}, Range[10], ControlType → Setter},
 {{tc, "cos", "time component"}, {"cos", "sin"}, ControlType → RadioButton},
 {{T0, 1}, 0.1, 3, ControlType → VerticalSlider, ControlPlacement → Right},
 {{ρ, 1}, 0.1, 3, ControlType → VerticalSlider, ControlPlacement → Left},
 {{lL, 1, "length"}, 0.1, 2, ControlType → Slider, ControlPlacement → Bottom}, ContinuousAction → True]
```

Out[9]=



In[10]:=
```
Module[{t}, t = 0;
 Plot[Evaluate[NMVC[x, t, 6, 1, 1, Pi]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
  Epilog → {
    {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
    {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
     Text[NumberForm[N[2 t], {3, 1}], {Pi / 2, 1.26},
      BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
   },
  PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}}, AspectRatio → 1 / 5, Frame → True,
  FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}}, Axes → False, ImageSize → 600, Background → Black]]
```

Out[10]=



Explore how the command Table[] works with nested tables.

In[11]:=
```
Table[Table[{j, k}, {j, 1, 4}], {k, 4, 8}]
```

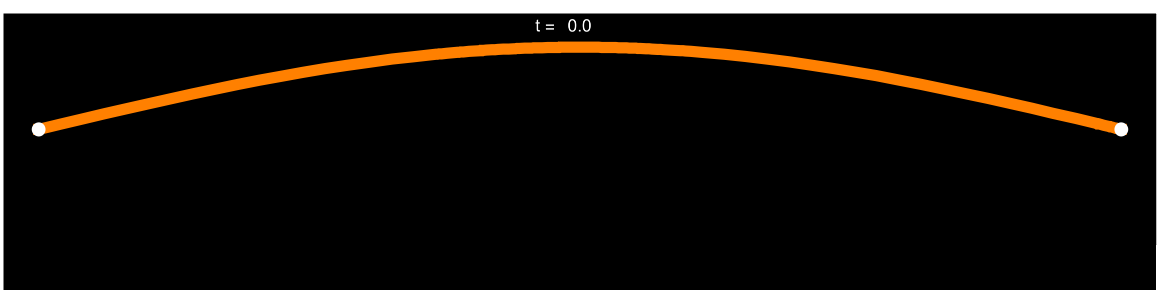
Out[11]=
```
{{{1, 4}, {2, 4}, {3, 4}, {4, 4}}, {{1, 5}, {2, 5}, {3, 5}, {4, 5}},
 {{1, 6}, {2, 6}, {3, 6}, {4, 6}}, {{1, 7}, {2, 7}, {3, 7}, {4, 7}}, {{1, 8}, {2, 8}, {3, 8}, {4, 8}}}
```

In[12]:=
```
NMV1 =
  Table[Plot[Evaluate[NMVC[x, t, 1, 1, 1, Pi]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
    Epilog → {
      {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
      {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
       Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.26},
        BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
     },
    PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}},
    AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
    Axes → False, ImageSize → 600, Background → Black], {t, 0, 2 Pi, 0.05}];
```

In[13]:=
```
Length[NMV1]
```

Out[13]= 126

In[14]:=
```
NMV1[[1]]
```

Out[14]=



In[15]:=
```
NMVC[x, t, n, 1, 1, Pi]
```

Out[15]=
```
Cos[n t] Sin[n x]
```

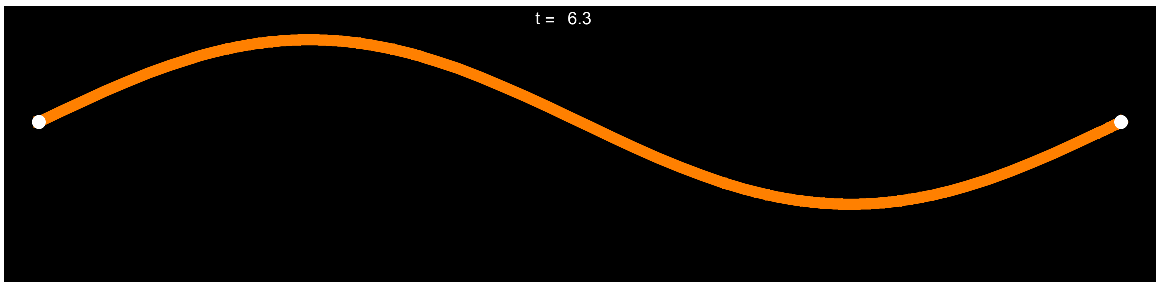
In[16]:=
```
NMVtt = Table[Table[
    Plot[Evaluate[NMVC[x, t, n, 1, 1, Pi]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
     Epilog → {
       {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
       {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
        Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.26},
         BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
      },
     PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}},
     AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
     Axes → False, ImageSize → 600, Background → Black], {t, 0, 2 Pi, 2 Pi / 120.}], {n, 1, 6}];
```

In[17]:=
```
Length[NMVtt⟦2⟧]
```

Out[17]= 121

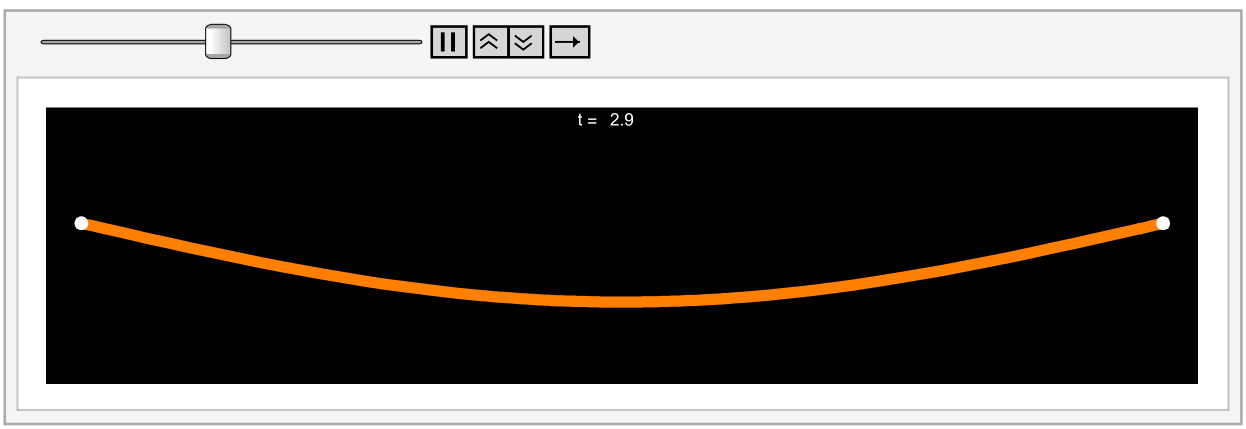
In[18]:=
```
Show[NMVtt⟦2, 121⟧, ImageSize → 600]
```

Out[18]=



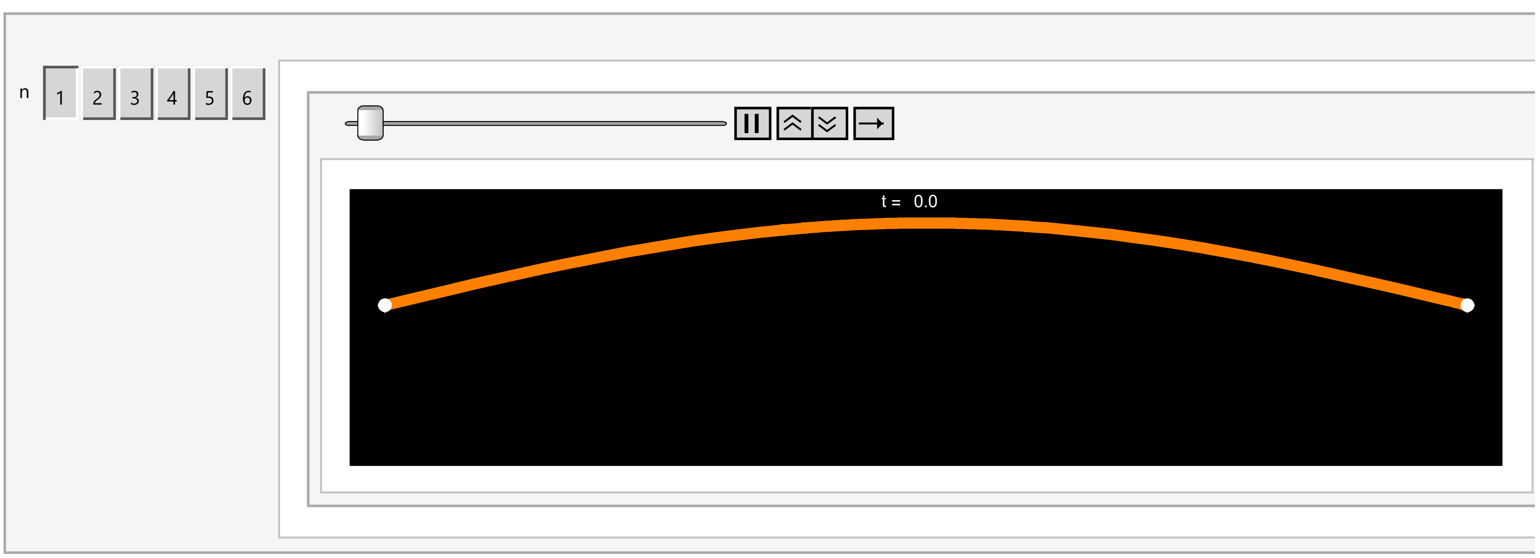

In[19]:=
```
ListAnimate[Show[#, ImageSize → 600] & /@ NMV1, ControlPlacement → Top]
```

Out[19]=




In[20]:=
```
Manipulate[ListAnimate[Show[#, ImageSize → 600] & /@ NMVtt⟦n⟧, AnimationRunning → True, ControlPlacement -> Top],
  {n, Range[1, 6]}, ControlType → Setter]
```

Out[20]=

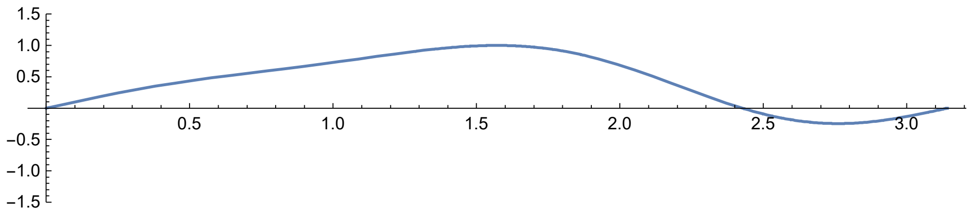


# An exact solution

I will use the function below as the initial displacement of the string.

In[21]:=
```
Plot[1 Sin[x]^3 + Cos[x]^3 * Sin[x]^1, {x, 0, Pi}, PlotRange → {-1.5, 1.5}, AspectRatio → 1 / 5, ImageSize → 500]
```

Out[21]=




In[22]:=
```
Table[2/Pi Integrate[# Sin[k x], {x, 0, Pi}], {k, 1, 12}] &[1 Sin[x]^3 + Cos[x]^3 * Sin[x]^1]
```

Out[22]= $\left\{\frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{8}, 0, 0, 0, 0, 0, 0, 0, 0\right\}$

In[23]:=
```
(Table[2/Pi Integrate[# Sin[k x], {x, 0, Pi}], {k, 1, 8}] &[1 Sin[x]^3 + Cos[x]^3 * Sin[x]^1]).
 Table[Sin[k x] Cos[k t], {k, 1, 8}]
```

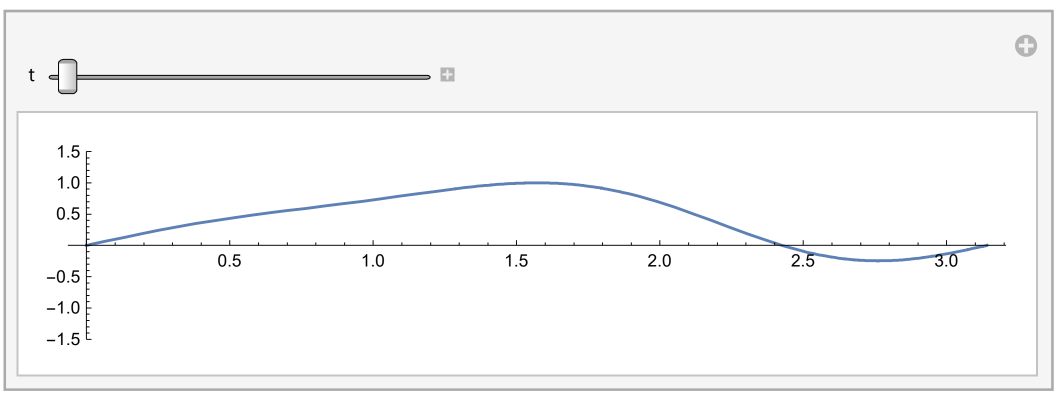
Out[23]= $\frac{3}{4} \mathrm{Cos}[t]\, \mathrm{Sin}[x] + \frac{1}{4} \mathrm{Cos}[2 t]\, \mathrm{Sin}[2 x] - \frac{1}{4} \mathrm{Cos}[3 t]\, \mathrm{Sin}[3 x] + \frac{1}{8} \mathrm{Cos}[4 t]\, \mathrm{Sin}[4 x]$

In[24]:=
```
FullSimplify[(%) /. {t → 0}]
```

Out[24]= $\frac{1}{8} (6 \mathrm{Sin}[x] + 2 \mathrm{Sin}[2 x] - 2 \mathrm{Sin}[3 x] + \mathrm{Sin}[4 x])$

In[25]:=
```
Manipulate[Plot[3/4 Cos[t] Sin[x] + 1/4 Cos[2 t] Sin[2 x] - 1/4 Cos[3 t] Sin[3 x] + 1/8 Cos[4 t] Sin[4 x], {x, 0, Pi},
  PlotRange → {-1.5, 1.5}, AspectRatio → 1 / 5, ImageSize → 500], {t, 0, 2 Pi}, ControlPlacement → Top]
```

Out[25]=



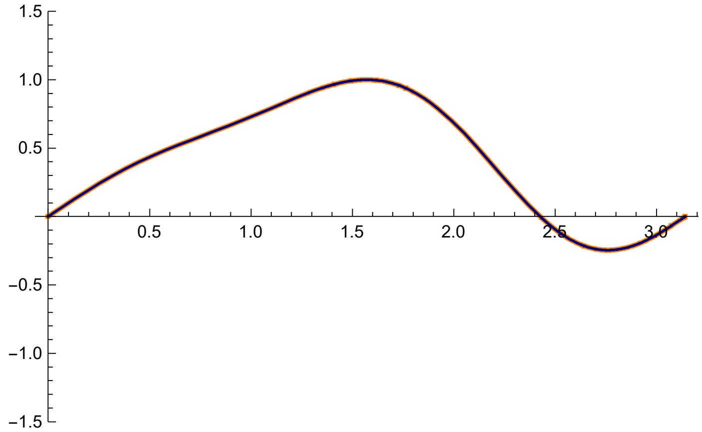

Since we will take the initial velocity to be zero, we only need the following separated solutions:

In[26]:=
```
uupex[x_, t_] = 3/4 Cos[t] Sin[x] + 1/4 Cos[2 t] Sin[2 x] - 1/4 Cos[3 t] Sin[3 x] + 1/8 Cos[4 t] Sin[4 x];
```

Just to verify, as the plot below shows. The orange function is the given function, and the navy function is a part of the solution uupex[x,t].

In[27]:=
```
Plot[{Sin[x]^3 + Cos[x]^3 * Sin[x]^1, Evaluate[uupex[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {-1.5, 1.5}]
```

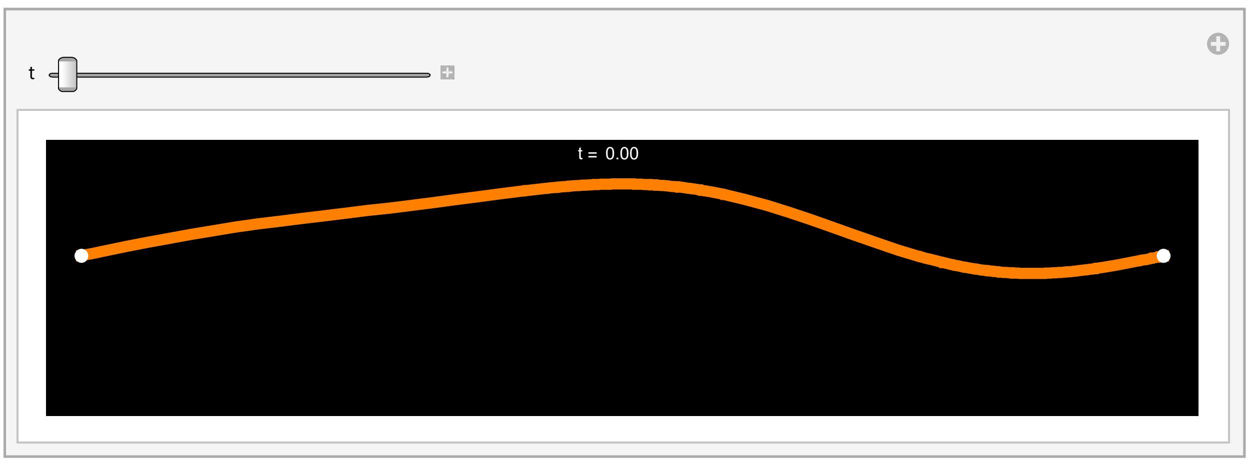
Out[27]=




Now show vibrations of this string.

In[28]:=
```
Manipulate[Plot[Evaluate[uupex[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
  Epilog → {
    {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
    {Text["t =", {Pi / 2 - 0.1, 1.42}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
     Text[NumberForm[N[t], {4, 2}], {Pi / 2, 1.42},
      BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
   },
  PlotRange → {{-0.1, Pi + 0.1}, {-1.6, 1.6}}, AspectRatio → 1 / 5,
  Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}}, Axes → False,
  ImageSize → 600, Background → Black], {t, 0, 2 Pi}, ControlPlacement → Top]
```

Out[28]=



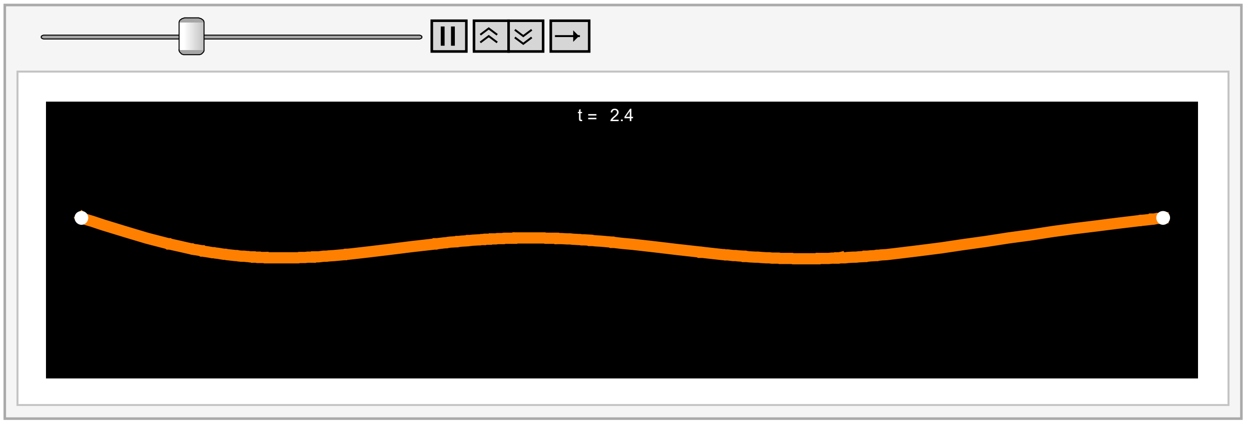

In[29]:=
```
uupextt = Table[Plot[Evaluate[uupex[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
    Epilog → {
      {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
      {Text["t =", {Pi / 2 - 0.1, 1.42}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
       Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.42},
        BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
     },
    PlotRange → {{-0.1, Pi + 0.1}, {-1.6, 1.6}},
    AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
    Axes → False, ImageSize → 800, Background → Black], {t, 0, 2 Pi, 2 Pi / 240}];
```

In[30]:=
```
Length[uupextt]
```

Out[30]= 241

In[31]:=
```
ListAnimate[Show[#, ImageSize → 600] & /@ uupextt, ControlPlacement → Top]
```

Out[31]=



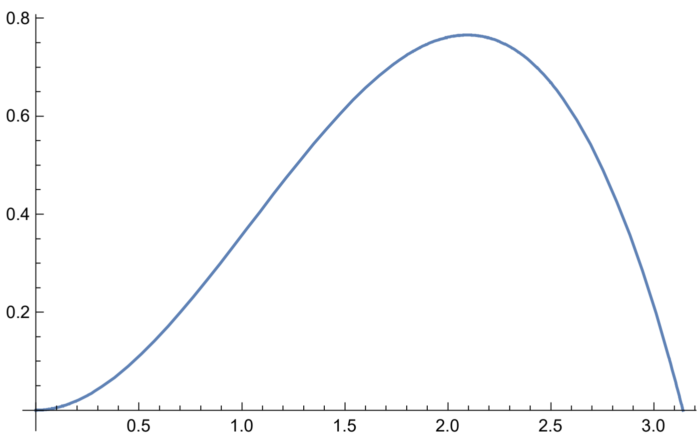

# Approximate solution using a partial sum of a Fourier series

## Exact formulas for the Fourier coefficients

I will use the function below as the initial displacement of the string.

In[32]:= $\texttt{Plot}\left[\frac{1}{6} x^2 (\texttt{Pi} - x), \{x, 0, \texttt{Pi}\}, \texttt{PlotRange} \to \texttt{All}\right]$

Out[32]=




Since we will take the initial velocity to be zero, we only need the following separated solutions:

In[33]:=
```
NMVC[x, t, n, 1, 1, Pi]
```

Out[33]=
```
Cos[n t] Sin[n x]
```

We will work with the approximation obtained with 50 terms of the corresponding Fourier series which converges uniformly to the initial condition.

In[34]:= $\texttt{FullSimplify}\left[\frac{2}{\texttt{Pi}} \texttt{Integrate}\left[\frac{1}{6} x^2 (\texttt{Pi} - x) \texttt{Sin}[n x], \{x, 0, \texttt{Pi}\}\right], \texttt{And}[n \in \texttt{Integers}, n > 0]\right]$

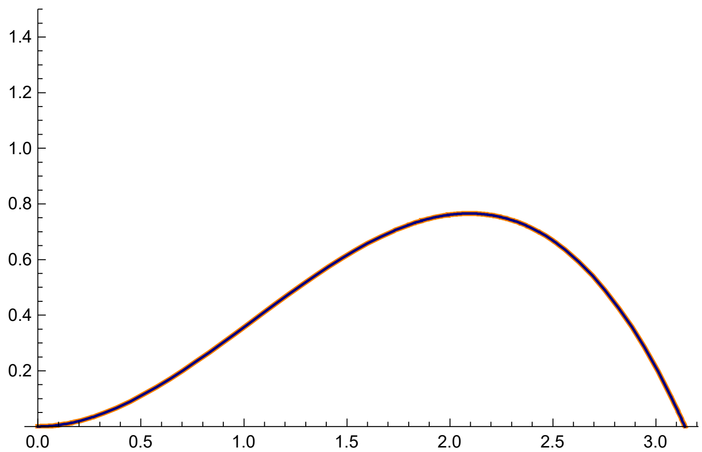
Out[34]= $-\frac{2\left(1 + 2 (-1)^n\right)}{3 n^3}$

In[35]:= $\texttt{uupef}[x\_, t\_] = \texttt{Sum}\left[-\frac{2\left(1 + 2 (-1)^n\right)}{3 n^3} \texttt{Cos}[n t] \texttt{Sin}[n x], \{n, 1, 50\}\right];$

We obtained a very good approximation, as the plot below shows. The orange function is the given function, and the navy function is the approximation.

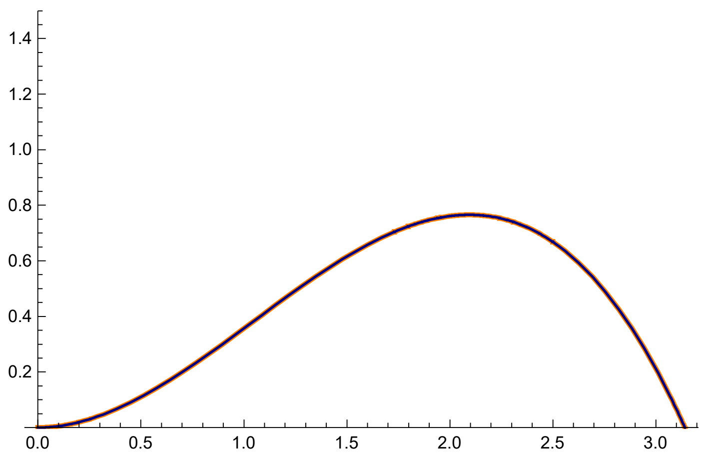
In[36]:=
```
Plot[{1/6 x^2 (Pi - x), Evaluate[uupef[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {0, 1.5}]
```

Out[36]=



In[37]:=
```
Plot[{Evaluate[uupef[x, 2 Pi]], Evaluate[uupef[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {0, 1.5}]
```

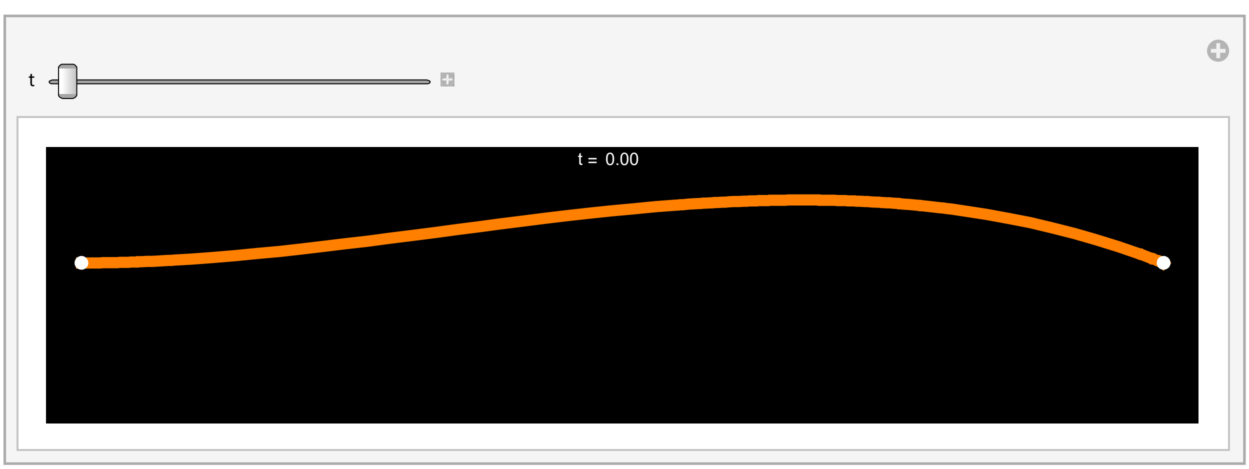
Out[37]=



Now show vibrations of this string.

In[38]:=
```
Manipulate[Plot[Evaluate[uupef[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
  Epilog → {
    {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
    {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
     Text[NumberForm[N[t], {4, 2}], {Pi / 2, 1.26},
      BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
   },
  PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}}, AspectRatio → 1 / 5,
  Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}}, Axes → False,
  ImageSize → 600, Background → Black], {t, 0, 2 Pi}, ControlPlacement → Top]
```

Out[38]=

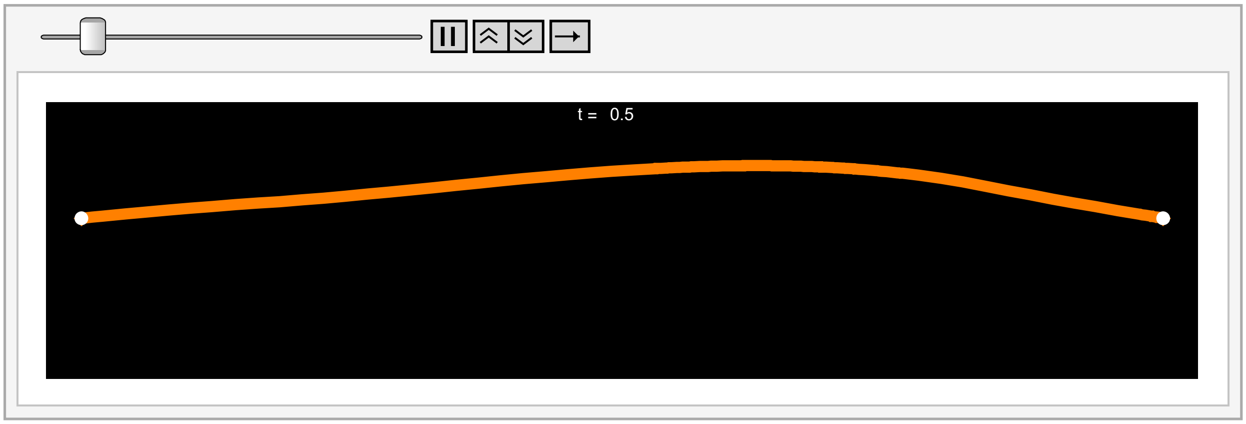


In[39]:=
```
uupeftt = Table[Plot[Evaluate[uupef[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
    Epilog → {
      {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
      {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
       Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.26},
        BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
     },
    PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}},
    AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
    Axes → False, ImageSize → 800, Background → Black], {t, 0, 6.4, 6.4 / 240}];
```

In[40]:=
```
Length[uupeftt]
```

Out[40]= 241

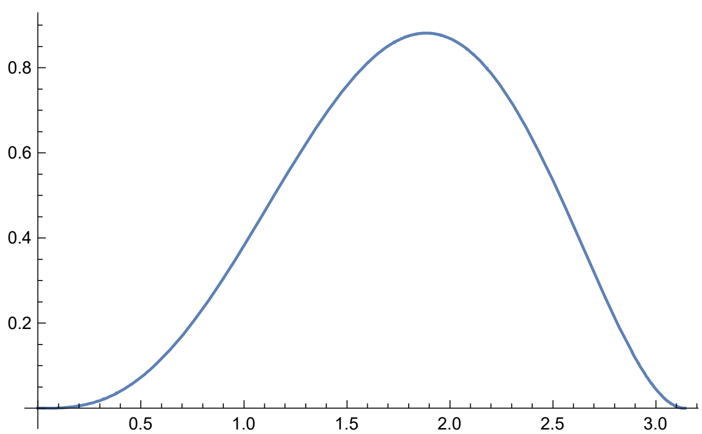
In[41]:=
```
ListAnimate[Show[#, ImageSize → 600] & /@ uupeftt, ControlPlacement → Top]
```

Out[41]=



## Exact formulas for the Fourier coefficients 2

I will use the function below as the initial displacement of the string.

In[42]:=
```
Plot[1/12 x^3 (Pi - x)^2, {x, 0, Pi}, PlotRange → All]
```

Out[42]=



Since we will take the initial velocity to be zero, we only need the following separated solutions:

In[43]:=
```
NMVC[x, t, n, 1, 1, Pi]
```

Out[43]= Cos[n t] Sin[n x]

We will work with the approximation obtained with 50 terms of the corresponding Fourier series which converges uniformly to the initial condition.

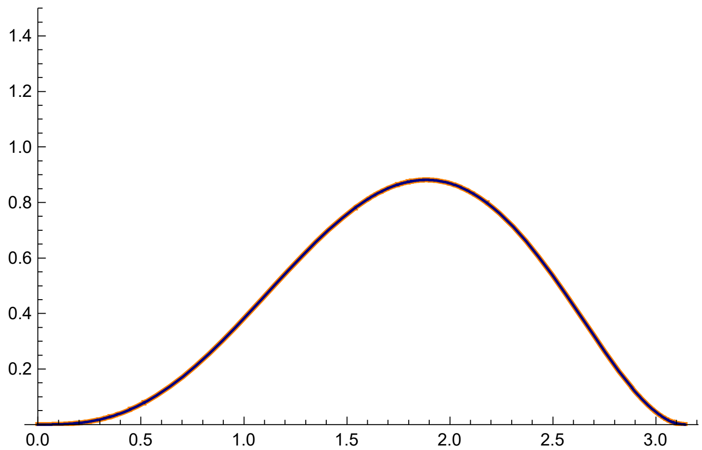
In[44]:= 
```
FullSimplify[2/Pi Integrate[1/12 x^3 (Pi - x)^2 Sin[n x], {x, 0, Pi}], And[n ∈ Integers, n > 0]]
```

Out[44]= $\frac{-24 + (-1)^n \left(-36 + n^2 \pi^2\right)}{3 n^5}$

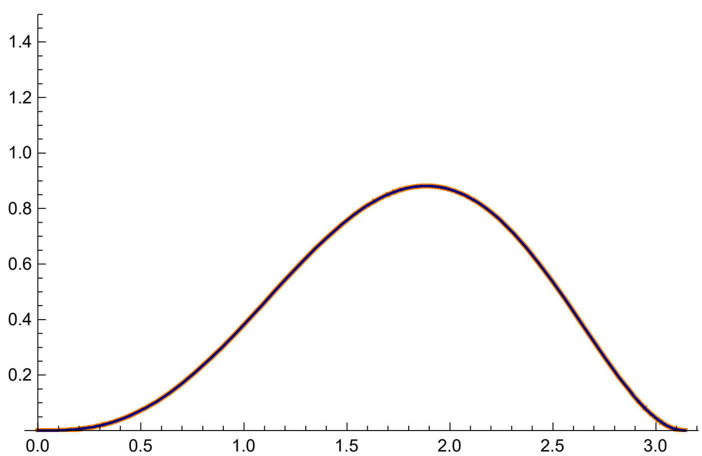
In[45]:= 
```
uupef2[x_, t_] = Sum[(-24 + (-1)^n (-36 + n^2 π^2))/(3 n^5) Cos[n t] Sin[n x], {n, 1, 50}];
```

We obtained a very good approximation, as the plot below shows. The orange function is the given function, and the navy function is the approximation.

In[46]:= 
```
Plot[{1/12 x^3 (Pi - x)^2, Evaluate[uupef2[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {0, 1.5}]
```

Out[46]= 


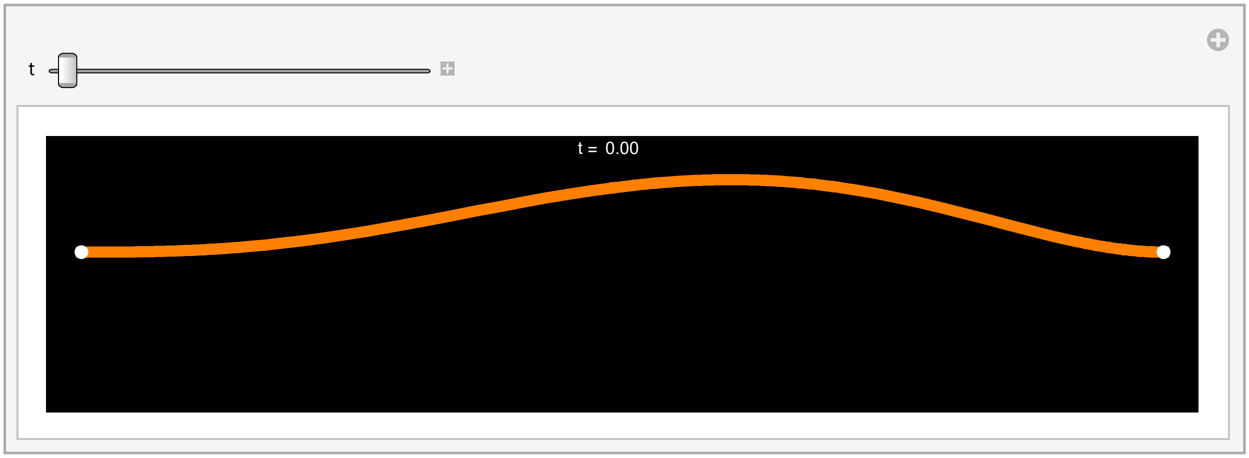

In[47]:= 
```
Plot[{Evaluate[uupef2[x, 2 Pi]], Evaluate[uupef2[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {0, 1.5}]
```

Out[47]= 



Now show vibrations of this string.

In[48]:=
```
Manipulate[Plot[Evaluate[uupef2[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
  Epilog → {
    {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
    {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
     Text[NumberForm[N[t], {4, 2}], {Pi / 2, 1.26},
      BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
   },
  PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}}, AspectRatio → 1 / 5,
  Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}}, Axes → False,
  ImageSize → 600, Background → Black], {t, 0, 2 Pi}, ControlPlacement → Top]
```

Out[48]=



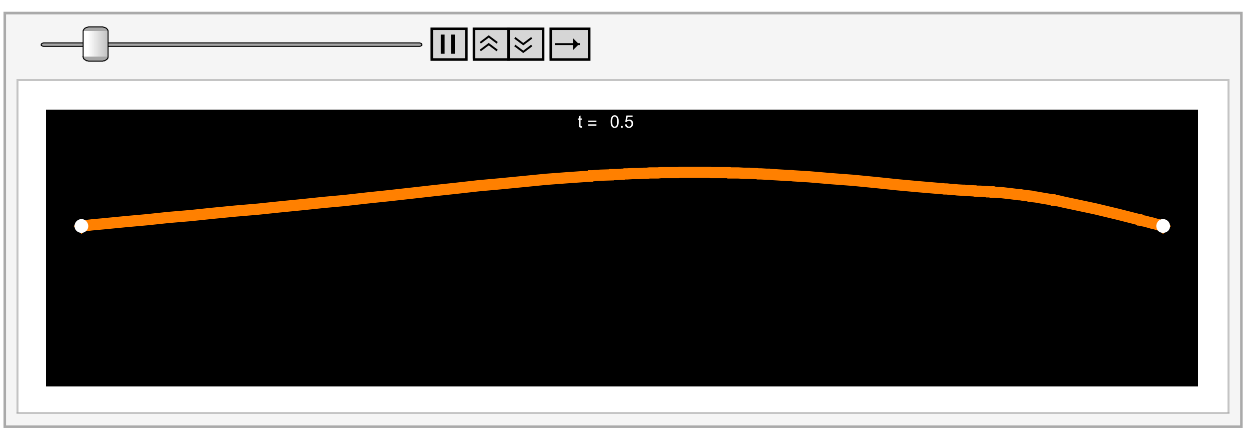

In[49]:=
```
uupeft2 = Table[Plot[Evaluate[uupef2[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
    Epilog → {
      {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
      {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
       Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.26},
        BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
     },
    PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}},
    AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
    Axes → False, ImageSize → 800, Background → Black], {t, 0, 2 Pi, 2 Pi / 240}];
```

In[50]:=
```
Length[uupeft2]
```

Out[50]= 241

In[51]:=
```
ListAnimate[Show[#, ImageSize → 600] & /@ uupeft2, ControlPlacement → Top]
```

Out[51]=

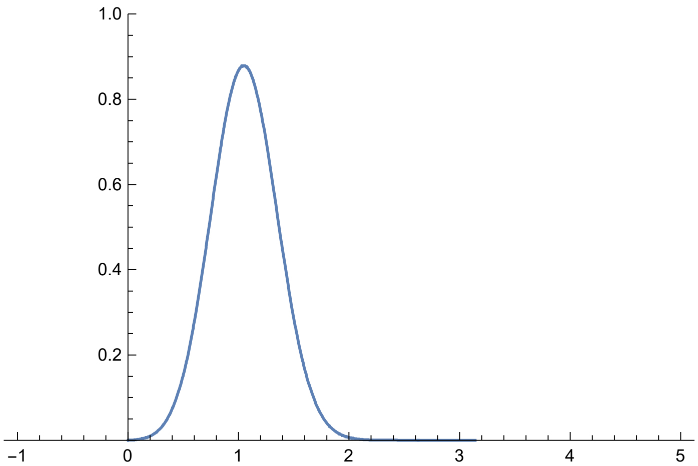



## Numerical evaluations for the Fourier coefficients

I will use the function below as the initial displacement of the string.

In[52]:=
```
Plot[4/Pi^2 x (Pi - x) Exp[-5 (x - 1)^2], {x, -1, 5}, PlotRange → {0, 1}]
```

Out[52]=



Since we will take the initial velocity to be zero, we only need the following separated solutions:

In[53]:=
```
NMVC[x, t, n, 1, 1, Pi]
```

Out[53]=
```
Cos[n t] Sin[n x]
```

We will work with the approximation obtained with 50 terms of the corresponding Fourier series which converges uniformly to the initial condition.

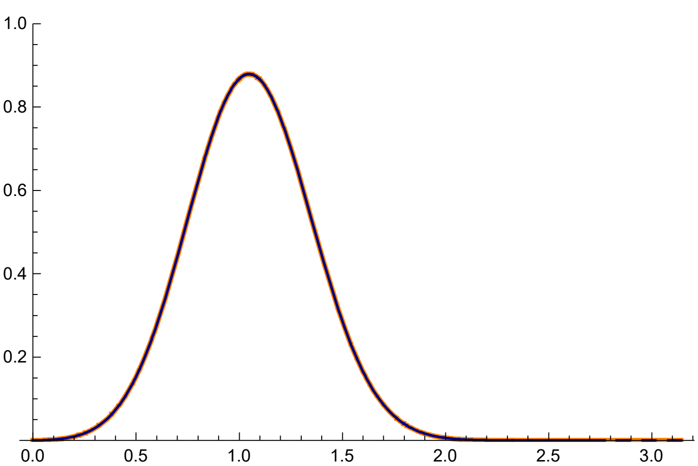
In[54]:=
```
Clear[aat]; aat = Table[2/Pi NIntegrate[4/Pi^2 x (Pi - x) Exp[-5 (x - 1)^2] Sin[n x], {x, 0, Pi}], {n, 1, 50}];
```

In[55]:=
```
uup[x_, t_] = Sum[aat[[n]] Cos[n t] Sin[n x], {n, 1, Length[aat]}];
```

We obtained a very good approximation, as the plot below shows. The orange function is the given function, and the navy function is the approximation.

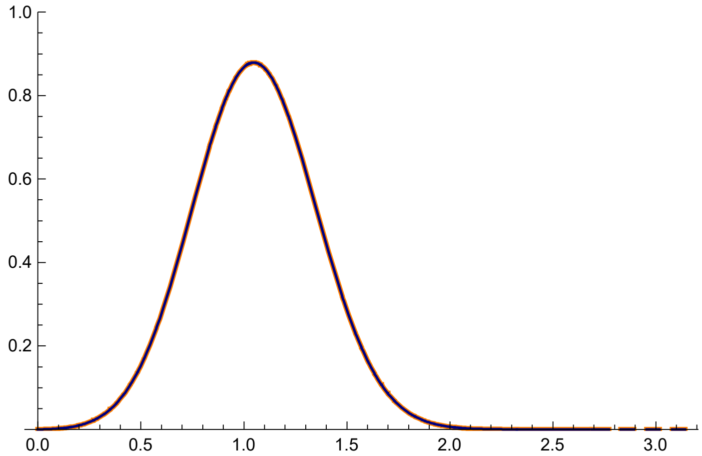
In[56]:=
```
Plot[{4/Pi^2 x (Pi - x) Exp[-5 (x - 1)^2], Evaluate[uup[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {0, 1}]
```

Out[56]=



In[57]:=
```
Plot[{Evaluate[uup[x, 2 Pi]], Evaluate[uup[x, 0]]}, {x, 0, Pi}, PlotStyle →
  {{Thickness[0.008], RGBColor[1, 0.5, 0]}, {Thickness[0.004], RGBColor[0, 0, 0.5]}}, PlotRange → {0, 1}]
```

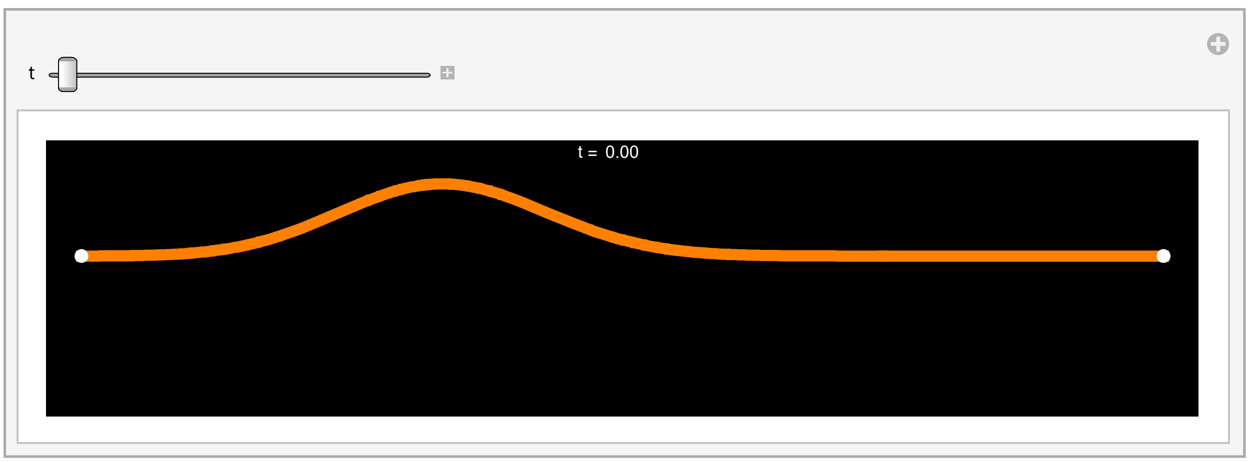
Out[57]=




Now show vibrations of this string.

In[58]:=
```
Manipulate[Plot[Evaluate[uup[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
  Epilog → {
    {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
    {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
     Text[NumberForm[N[t], {4, 2}], {Pi / 2, 1.26},
      BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
   },
  PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}}, AspectRatio → 1 / 5,
  Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}}, Axes → False,
  ImageSize → 600, Background → Black], {t, 0, 2 Pi}, ControlPlacement → Top]
```

Out[58]=



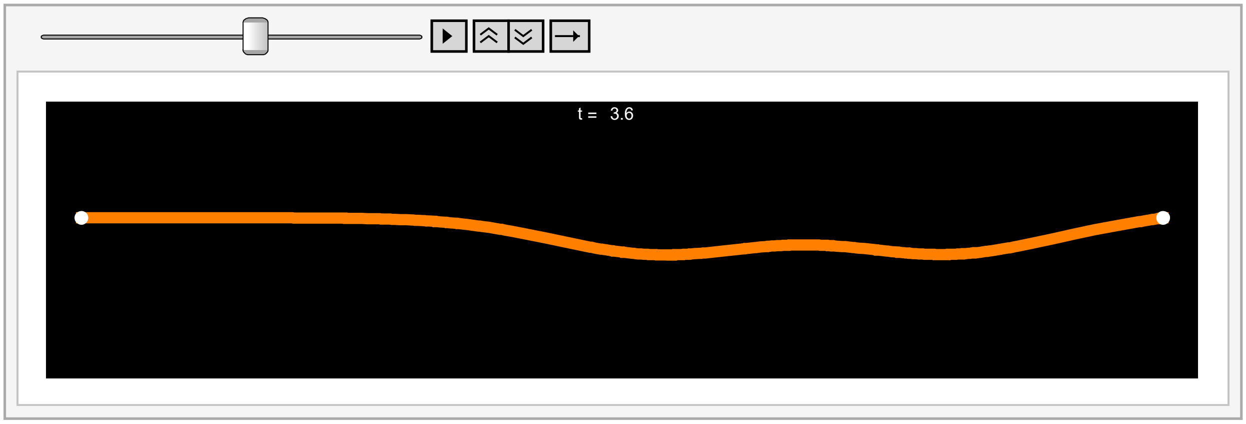

In[59]:=
```
uuptt = Table[Plot[Evaluate[uup[x, t]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
    Epilog → {
      {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
      {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
       Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.26},
        BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
     },
    PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}},
    AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
    Axes → False, ImageSize → 800, Background → Black], {t, 0, 2 Pi, 2 Pi / 240}];
```

In[60]:=
```
Length[uuptt]
```

Out[60]= 241

In[61]:=
```
ListAnimate[Show[#, ImageSize → 600] & /@ uuptt, ControlPlacement → Top]
```

Out[61]=




# Making animated gifs and pngs

The commands below is how I make animated gifs in Mathematica. I forst create a table of pictures. Then export those tables as gifs. That creates an animated gifs. The modern browsers can also display animated pngs.

In[62]:=
```
NMVttime = Table[Table[
    Plot[Evaluate[NMVC[x, t, n, 1, 1, Pi]], {x, 0, Pi}, PlotStyle → {{Thickness[0.01], RGBColor[1, 0.5, 0]}},
     Epilog → {
       {PointSize[0.012], White, Point[#] & /@ {{0, 0}, {Pi, 0}}},
       {Text["t =", {Pi / 2 - 0.1, 1.26}, BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}],
        Text[NumberForm[N[t], {3, 1}], {Pi / 2, 1.26},
         BaseStyle → {FontWeight → "Normal", FontColor → RGBColor[1, 1, 1]}]}
      },
     PlotRange → {{-0.1, Pi + 0.1}, {-1.4, 1.4}},
     AspectRatio → 1 / 5, Frame → True, FrameTicks → {{{}, {}}, {Range[0, Pi, Pi / 4], {}}},
     Axes → False, ImageSize → 800, Background → Black], {t, 0, 2 Pi, 2 Pi / 240.}], {n, 1, 6}];
```

In[63]:=
```
Length[NMVttime[[1]]]
```

Out[63]= 241

In[64]:=
```
(* dds=0.1&/@Range[Length[NMVttime[[1]]]];  duration of each frame that we want*)
```

In[65]:=
```
NotebookDirectory[]
```

Out[65]= C:\Dropbox\Work\myweb\Courses\Math_pages\Math_430\

The commands below are commented out since I do not want them executed whenever I evaluate this notebook. The commands below export the tables created above as animated gifs. I hope I got them reasonable well designed, so until I decide that I can improve them, they will stay commented out.

In[66]:=
```
(* SetDirectory[NotebookDirectory[]] *)
```

In[67]:=
```
(* Export["NMVttime1s1.gif",NMVttime[[1]][[1]],"ImageSize"→800];
Export["NMVttime1.gif",NMVttime[[1]],"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ;
  Export["NMVttime2s1.gif",NMVttime[[2]][[1]],"ImageSize"→800];
Export["NMVttime2.gif",NMVttime[[2]],"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ;
Export["NMVttime3s1.gif",NMVttime[[3]][[1]],"ImageSize"→800];
Export["NMVttime3.gif",NMVttime[[3]],"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ;
Export["NMVttime4s1.gif",NMVttime[[4]][[1]],"ImageSize"→800];
Export["NMVttime4.gif",NMVttime[[4]],"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ;
Export["NMVttime5s1.gif",NMVttime[[5]][[1]],"ImageSize"→800];
Export["NMVttime5.gif",NMVttime[[5]],"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ;
Export["NMVttime6s1.gif",NMVttime[[6]][[1]],"ImageSize"→800];
Export["NMVttime6.gif",NMVttime[[6]],
 "AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds]  *)
```

In[68]:=
```
(* Export["uupts1.gif",uuptt[[1]],"ImageSize"→800];
Export["uupttAni.gif",uuptt,"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ; *)
```

In[69]:=
```
(*
SetDirectory[NotebookDirectory[]] ;
dds=0.1&/@Range[Length[uupeftt]];
Export["uupeftts1.gif",uupeftt[[1]],"ImageSize"→800];
Export["uupefttAni.gif",uupeftt,"AnimationRepetitions"→Infinity,"ImageSize"→800,"DisplayDurations"→dds] ;
*)
```